DEVELOPING SECOND GRADE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF PLACE VALUE

A Record of Study

by

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ABSTRACT

An understanding of whole number place value is a critical component of second-grade mathematics. This understanding of place value provides the foundational concept for operations with whole numbers. The ability to understand the concept of place value and transfer that understanding to teaching addition and subtraction are often problems associated with teachers' limited pedagogical content knowledge. Teachers must understand teaching place value does not consist of merely teaching students to name the digit in a particular location. A possible solution to the problem is to provide professional development trainings and implementation support through instructional coaching focused on the teaching of place value in a conceptual manner consistent with best pedagogical practices. Therefore, the purpose of this mixed method study is to describe the pedagogical content knowledge of second-grade teachers in order to answer the research question: What can you expect to see in the classroom teaching of place value from second-grade teachers who have attended 12 hours of professional development and have received instructional coaching?

Quantitative data includes the use of the *Learning Mathematics for Teaching* assessment paired with participant observations, discussions, and interviews in order to gain a deeper understanding of the participants' thinking. The participants of this study are four second-grade teachers from a rural school district outside of Houston, Texas. Each of the four participants attended 12 hours of professional development and

received instructional coaching support through classroom observation with post-observation discussions, a model lesson, and instructional planning.

The results of the study show that every participant implemented activities from the professional development, used questioning modeled by the presenter during the sessions, and discussed critical concepts in their classrooms. Additionally, teacher pedagogical content knowledge did improve after 12 hours of professional development and instructional coaching. However, the participants believe that further support should be given through a more detailed scope and sequence, which provides information as to when and how the new concepts and activities need to be incorporated in the classroom.

DEDICATION

To my husband and best friend, Kevin, who has walked beside me through many of life's journeys, including this endeavor. I could not have made this trip without your love and support.

To my beautiful twin girls, Makenzie and Payton, I hope that the struggles and sacrifices you have witnessed over these past four years will one day inspire you to work hard to achieve your dreams.

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CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

In 1983, The National Commission on Excellence in Education published the landmark report *A Nation at Risk: The Imperative for Educational Reform.* This report claimed that many teachers graduated in the bottom quarter of their high school or college graduating classes and, more specifically, half of the employed mathematics, science, and English teachers were not qualified to teach these subjects (National Commission on Excellence in Education, 1983). Despite these findings and associated recommendations, the U.S. educational system continued to struggle to reform and improve teacher quality. Among teachers in a study by Hill, Rowan, and Ball (2005), 12% reported never having taken a mathematics content course or a mathematics methods course and 15% reported taking only one to three of these courses. The limited opportunities for teachers to develop pedagogical content knowledge was a critical concern because research suggested teacher quality was a significant factor in predicting student achievement (Akiba, LeTendre, & Scribner, 2007; Hill et al., 2005). In fact, the National Mathematics Advisory Panel (NMAP, 2008) found that fourth- and eighth-grade U.S. students scored below students from several other countries in areas such as algebra. Some education leaders believed teachers' lack of conceptual understanding was one of the factors contributing to the United States' disappointing mathematical rank among countries worldwide (National Mathematics Advisory Panel [NMAP], 2008; Hill et al., 2005; Baumert et al., 2009).

In preparing the 21st-century learner for future careers not yet in existence, teachers were required to provide mathematics instruction that not only included procedural skills such as rote memorization but also placed an emphasis on problem solving and critical-thinking skills. Literature showed that teachers successfully performed computations; however, many were unable to provide conceptual explanations for these procedures (Mewborn, 2001). Problem solving involved engaging in a task or problem where a solution method was not readily available and using critical thinking skills of reasoning and/or generalizing to find the solution (National Council of Teachers of Mathematics [NCTM], 2000). Teachers could not facilitate students' understanding of critical thinking and problem solving without a deep understanding of their subject matter content. Additionally, teachers of mathematics needed to know how to use pictures or diagrams to represent mathematics concepts and procedures, explain mathematical rules, and analyze students' solutions and explanations (Hill et al., 2005). A teacher proficient at explaining, modeling, breaking concepts into smaller parts, and engaging students in problem solving and critical thinking was vital to students' mathematical understandings.

Conceptual understandings referred to an individual's ability to integrate mathematical ideas, represent mathematics in different ways, or use the knowledge of mathematics in a variety of situations (National Research Council [NRC], 2001). Some teachers had a conceptual understanding of mathematics but lacked pedagogical knowledge to instruct their students. Pedagogy, or teaching practices, included tasks such as classroom management, lesson planning, or being able to group and regroup

students into partners, small groups, whole group, etc., based on the needs of the student (Garet, Porter, Desimone, Berman, & Yoon, 2001). Furthermore, teachers needed the ability to apply their pedagogical knowledge to specific mathematical activities. For example, teachers needed to be able to choose meaningful mathematical activities that facilitate students' interactions about a mathematical concept and scaffold students' construction of mathematical ideas (Bruce & Ross, 2008). A teacher's mathematics content knowledge, unless accompanied by pedagogical knowledge of curriculum, instruction, and student learning, was ineffective (Baumert et al., 2009). Thus, teaching required teachers to possess pedagogical content knowledge unique to teaching mathematics (Ball, Thames, & Phelps, 2008) or the area of knowledge required to communicate mathematics to students (Baumert et al., 2009). Given the importance of pedagogical content knowledge and the fact that some teachers struggled with these skill sets in the discipline of mathematics, a conscious effort was necessary to develop and refine teachers' conceptual understandings and pedagogical capacities in critical mathematics content areas such as place value.

Problem

United States and Texas

As an elementary mathematics specialist, I have worked with numerous elementary teachers from various districts throughout the state of Texas. In the course of my work, I have found that many of these teachers lack in-depth understandings of mathematical concepts, have limited pedagogical knowledge of mathematics, or a combination of both, leading to a student's compromised ability to develop an

understanding place value. An understanding of place value is necessary for the successful completion of many states' required high school algebra courses. Algebraic concepts are vital for preparing students in the United States to lead this country in careers and technological advances. In fact, students who completed Algebra II were more than 50% more likely to graduate from college than a student who did not complete the course (NMAP, 2008). As a result, the Texas Education Agency implemented the Texas Algebra Ready Initiative to prepare every student in the state to take and pass Algebra II. The three critical foundations for algebra—fluency with whole numbers, fluency with fractions, and an understanding of certain aspects of geometry and measurement concepts (NMAP, 2008)—were guiding principles for the Initiative. In order to develop a conceptual understanding of these three critical foundations, one needed to manipulate whole numbers and fractions with computational fluency as well as apply these understandings to geometric and measurement principles in problem-solving situations.

This study specifically focused on the understanding of fluency with whole numbers, which included an understanding of place value. An understanding of place value with whole numbers and the ability to apply those understandings to solving addition and subtraction problems with whole numbers is a critical component of second-grade mathematics. By the end of third grade, students are expected to add and subtract whole numbers fluently and efficiently (NMAP, 2008). Therefore, teachers' pedagogical content knowledge of place value is necessary to assist their students in utilizing the concept as a strategy for solving operation problems (NCTM, 2010). The

foundations of whole number place value are also requisite skills for third-grade students who begin to transfer their whole number place value understandings to fractions and decimals concepts. Therefore, in order for students to apply place value understandings to operations, fractions, and decimals concepts, teachers are required to developing students' concept of place value in a manner that goes beyond asking students to identify and name the digit in a particular location. Place value instruction must require students to focus on the value of the digits and apply their understandings about the value of the digits to operations. Therefore, teachers are obligated to possess an understanding of place value that includes an understanding of number and how numbers relate to one another (Ashlock, 2009).

Local School District

During my doctoral internship, I became familiar with teachers' misconceptions of place value, or lack of pedagogical concept knowledge of place value. Data from the district's benchmark assessments indicated that second- and third-grade students did not have an understanding of two- or three-digit place value. The district's second-grade benchmark data showed approximately 40% of students were successful when asked to apply their understanding of place value on the assessment, which was similar to the approximately 40% of students who were successful on the place value items found on the third grade beginning of the year district benchmark. Additionally, during my second-grade classroom observations, teachers focused on identifying the digit in the hundreds place or tens place instead of explaining that the value of a digit in a written numeral depends on its place, or position, in a number (NCTM, 2010). For example,

teachers asked questions such as, "Which digit is in the hundreds place?" instead of asking, "What is the value of the digit in the hundreds place?" This line of thinking became problematic as students began to subtract 28 from 52. Students were not able to understand how a "5" became 4 tens and 10 ones when it was necessary to regroup 5 tens into 4 tens and 12 ones.

Through conversations with Dr. Susan Johnson, Mathematics Director and my internship supervisor, it was determined that teachers' ability to understand the concept of place value and then transfer those understandings to addition and subtraction concepts were problems associated with the deficit teacher knowledge in her school district. Therefore, the purpose of this study was to gain an understanding of the district's second-grade teachers' pedagogical content knowledge of place value. Furthermore, by examining the results of the solution strategies, professional development, and instructional coaching, this research allowed district leaders and other interested parties to gain an understanding of what to expect in regards to developing teachers' pedagogical content knowledge.

Purpose Statement

The purpose of this study was to understand and describe the professional development and instructional coaching experiences of four second-grade teachers. At this stage in the research, a content and pedagogical assessment, classroom observations, and teacher content knowledge data provided insight into teachers' pedagogical content knowledge of place value.

Central Research Question

What can you expect to see in the classroom teaching of place value from second-grade teachers who have attended 12 hours of professional development and have had instructional coaching?

Literature Review

Practical Solutions

Although current research on the need to develop teachers' concept of place value was scarce, a research study of preservice teachers by Thanheiser (2009) found these future teachers could not explain what was occurring with place value; they simply knew how to follow the procedures or rules. In a similar study by Philipp, Schappelle, Siegfried, Jacobs, and Lamb (2008), prospective elementary teachers struggled to explain the differing representations of "1" when using the subtraction algorithm. The need for developing this concept was important because teachers must be able to simultaneously connect place value procedures and understandings with the students' knowledge of operations (Russell, 2000).

Based on a review of the literature, a practical solution to this problem was to offer second-grade teachers the opportunity to attend professional development sessions focused on teaching place value in a conceptual manner consistent with best pedagogical practices. These experiences aligned with research recommendations that suggested the professional development workshops should focus on content knowledge, emphasize active learning, promote coherence, and encourage collaboration (Garet et al., 2001). In conjunction with the professional development, instructional coaching provided

additional opportunities to increase teachers' pedagogical content knowledge of place value with whole numbers. Additional support through instructional coaching provided a sustained impact on teacher practice (Garet et al., 2001; Obara, 2010). Furthermore, combining instructional coaching with content-based professional development sessions provided a vehicle to further increase teachers' understanding by offering them the possibility of developing deep pedagogical knowledge and specific content understandings more fully (Bruce & Ross, 2008).

Specifically, based on several research studies, the professional development trainings and instructional coaching included a focus on student thinking in order to improve the teachers' pedagogical content knowledge. In a longitudinal study, Franke, Carpenter, Levi, and Fennema (2001) examined the sustainability of a professional development training four years after the teachers' completion of the initial professional development sessions. The study characterized factors that supported teachers' professional growth. The authors found that when teachers applied students' mathematical understandings to their own understandings, they were able to create generative change, which was the ability to continue to add to one's understandings. This information helped solidify the fact that professional development opportunities could focus on students' thinking in order to achieve desired levels of generative change (Franke, Carpenter, Levi, & Fennema, 2001). Studies showed that as teachers in a professional development setting sought to understand how children think about and develop mathematical understandings, they themselves learned mathematical concepts (Philipp, Schappelle, Siegfried, Jacobs, & Lamb, 2008; Elmore, 2002). Additionally,

Bruce and Ross (2008) found instructional coaching should include activities such as examining student work and discussing how students learned mathematics to help develop teachers' pedagogical content knowledge.

CHAPTER II

METHODS AND RESULTS

Rationale for Mixed Methods

In order to determine what one can expect to see in the classroom teaching of place value from second-grade teachers who have attended 12 hours of professional development and have had instructional coaching, I conducted a mixed method study, as described by Greene and Caracelli (2003). For the purpose of this study, quantitative and qualitative methods together helped facilitate a broader and deeper understanding of teachers' current pedagogical content knowledge (Greene & Caracelli, 2003). Additionally, the mixed methods helped assess whether or not the solution, the professional learning opportunities, had been effective. Together, the quantitative and qualitative data triangulation provided more data and answered a wider range of questions through diverse ways of thinking, knowing, and valuing, which allowed for a deeper understanding of teachers' pedagogical content knowledge (Greene & Caracelli, 2003).

All researchers' inquiries need a philosophical framework to guide their design and implementation. Underlying assumptions differ for philosophical frameworks that reflect the underlying ideas and beliefs about the role of the research study in understanding a particular educational context. Researchers' intentions to do research are deeply rooted in their own personal experiences, their culture, and their history. A researcher needs to be able to articulate and defend choices about the research methods

chosen to conduct research. To defend methods, the researcher must be familiar with three predominant worldviews most important in field-based studies: quantitative, qualitative, and mixed method approaches. The researcher must also be able to single out mixed methods as most appropriate, due to its pragmatic (practical) approach. I used the mixed methods approach because it was the most appropriate approach in terms of the field-based context and question I had chosen to investigate in my record of study. This study embedded information about two other predominant worldviews (i.e., quantitative and qualitative) to convince the reader that mixed methods was indeed the most appropriate approach for this field-based research study (Creswell & Plano Clark, 2007). The context and central question was most appropriate for a mixed approach because I was interested in knowing more about a small group of second grade teachers' pedagogical content knowledge of place value.

Data Collection

Quantitative Methods

Quantitative approaches in my mixed methods study included collecting data about the teachers' understanding using the Learning Mathematics for Teaching (LMTP, 2006) pretest and posttest from the University of Michigan. All of the 27 teachers who attended the first day of the professional development training in August 2012, completed the *Learning Mathematics for Teaching Survey of Elementary Teachers of Mathematics: Number Concepts and Operations Study* (LMTP, 2006) pretest. Figure 1 provides sample items from the LMT assessment (Hill, Schilling, & Ball, 2004).

Figure 1 Sample items from LMT assessment

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

983
x 6
488
+5410
5898

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

Studies on the LMT showed positive evidence that teachers' scores predicted teachers' mathematical instruction and student learning from that instruction; thus, making the assessment a valuable tool in the data collection process (Hill, Ball, Blunk, Goffney, & Rowan, 2007). During pilot testing of elementary teachers, the LMT produced adequate reliabilities (.70 or above) with samples of 60 or more participants (Hill et al., 2004). The use of the LMT demonstrated how the group of teachers performed at the time of the pretest and posttest but did not make claims about an individual teacher's understanding of the content (Hill et al., 2004). For the purpose of this study, questions pertaining to place value (whole numbers or decimal) and operations (addition, subtraction, multiplication, division) concepts and pedagogy provided insight into teachers' pedagogical content knowledge of place value. However, this quantitative data alone was not sufficient for answering the research question in my proposed study because the data from multiple choice assessment items alone did not provide insight into the teachers' ability to apply their understandings of place value in the classroom.

Qualitative Methods

Qualitative data enhanced my mixed methods study by allowing me to know more about the results of the teachers' LMT scores. Specifically, data gained from the classroom instructional coaching, the post-observation teacher interviews found in Appendix C, and the semistructured interview protocol found in Appendix D provided evidence for my research question. The classroom instructional coaching included observations, model lessons, and helping teachers plan upcoming lessons or activities.

These activities provided information about the activities, concepts, and pedagogy the teachers used in order to teach place value. This also allowed me to determine whether the teachers used ideas from the professional development training in the classroom or in their planning for mathematics instruction. The follow-up conversation, or post-observation conference, allowed me to ask the teachers questions to determine why they made certain conceptual and pedagogical decisions for that particular lesson. Furthermore, the semistructured interview, completed after the post-LMT, allowed the participant to think aloud and describe thoughts about place value. However, qualitative data alone was not sufficient either for answering the research question in my proposed study because it did not provide access to valid and reliable measures found in the LMT.

Mixed Methods

Giving up the strict assumptions of either quantitative or qualitative approaches, I therefore adopted a pragmatic approach. The pragmatic approach provided me with the flexibility to collect, analyze, and synthesize data to yield more knowledge about the pedagogical content knowledge of place value of second-grade teachers in a local district, than by adopting either one of the two stricter approaches would allow (Creswell & Plano Clark, 2007). I understand that I gave up generalizability, a major goal in quantitative methods, in order to understand particular aspects of this particular context in more depth.

Context

The participants for this study were from a population of approximately 50 second-grade teachers from a local, rural school district 45 miles northwest of Houston,

Texas. According to the 2010–2011 Academic Excellence Indicator System (AEIS), the district has a student population comprised of approximately 69% White, 26% Hispanic, and 2% African American with 11% Limited English Proficient, 41% economically disadvantaged, and 36% at-risk (Texas Education Agency [TEA], 2011). The district consists of eight elementary campuses that serve approximately 850 second-grade students; each second-grade classroom has an approximate average class size of 19 students. Based on the same AEIS report, the average years of teaching experience is 12.4 years; therefore, generally, the teachers in the district are experienced (TEA, 2011), yet they often teach place value through procedural memorization strategies instead of developing the concept of place value.

Participant Selection

Every kindergarten through third grade teacher in the district being studied received an invitation to attend 12 hours of focused professional development in August 2012. The professional development provided specific pedagogical and content knowledge to develop teachers' understanding of place value. Before experiencing the professional development activities, all of the 27 participants from kindergarten through third grade completed the LMT. The mean of the LMT responses for questions pertaining to place value and operations for this group of teachers was 0.53. For this group of participants, the mode was 0.48 and median was 0.50. From the population data, the second-grade teachers' scores were extrapolated in order to find two second-grade teachers scoring above the population mean and two second-grade teachers scoring below the mean on the LMT assessment. Based on the LMT data shown in

Table 1, these four second grade teachers become the subjects of this study. In order to protect the participants' anonymity, the four participants were referred to as Participant A, Participant B, Participant C, and Participant D throughout the study.

Table 1 LMT results

Participant	Score	+/– Mean
A	0.56	+ 0.03
B	0.58	+ 0.05
C	0.48	– 0.05
D	0.41	– 0.12

Study Participants

Table 2 shows the demographic information of the four participating teachers: ethnicity and the number of years each participant has been in education.

Table 2 Participant demographic information

Participant	Ethnicity	Gender	Years in Education
A	White	Female	12
B	White	Female	16
C	White	Female	10
D	White	Female	18

Based on the small sample size, the opportunities for diverse ethnicities, gender, and educational experience of those second-grade teachers attending 12 hours of professional develop were limited.

Participant A

Participant A teaches on a Title I campus, which failed to meet Adequately Yearly Progress (AYP) for both mathematics and reading. Last year, Participant A's second grade team was departmentalized and she was a reading/language arts teachers. This year, all of the

second-grade teachers are self-contained in an effort to increase test scores on the campus. Participant A completed three college mathematics content course and one mathematics methods courses. She was one of the teachers who scored above the population mean on the LMT pretest. Participant A was described by a campus administrator as a highly requested teacher in the community; employees' children attending school on the campus are placed in her second-grade classroom.

Participant B

Participant B also teaches second grade on a Title I campus, which failed to meet AYP for mathematics. For the past three years, Participant B taught third-grade language arts. She completed college algebra and one math methods course. Participant B was also one of the teachers who scored above the population mean on the LMT pretest.

Participant C

Participant C teaches second grade on the same campus, which failed to meet AYP for reading. She completed an algebra course and one mathematics methods course

in college. Participant C scored below the population mean on the LMT pretest. Similar to Participant A, this teacher was considered one of the best by fellow teachers as well as parents.

Participant D

Participant D teaches on the same campus as Participant C, the campus that failed to meet AYP for reading. She completed two college-level content courses and one mathematics methods course. Out of the four participants, Participant D received the lowest LMT pretest score. Participant D was overheard by her administrator at the professional development saying, "I already do this."

Solution

Professional Development

The ideas, findings, and experiences of several mathematicians concerning place value informed the design and content of the two-day professional development sessions and instructional coaching. For example, in a comprehensive study, Ross (1986) found young students were able to determine the quantity of 25 sticks and write the number, but it was not until fourth grade that students were able to explain that the digit 5 represented five sticks and the digit 2 represented 20 sticks. Based on the Texas Essential Knowledge and Skills for mathematics (TEA, 2012) and other standards such as the Common Core (2012), students need this understanding in second grade. The professional development provided suggestions and instructional activities to help teachers facilitate students in building this concept and understanding.

In order to provide foundations for such pedagogical content knowledge presented during the professional development trainings, the concepts and recommendations of the National Council of Teachers of Mathematics' (2010) *Developing Essential Understandings of Number and Numeration for Teaching Mathematics in Pre-K—2* and Catherine Fosnot and Maarten Dolk's (2001) *Constructing Number Sense, Addition, and Subtraction* were reviewed. Some of the key understandings of place value from the training included

- components of numerical fluency: one-to-one correspondence, subitizing, more than, less than, greater than, unitizing, part-part-whole, inclusion of set, etc. (NCTM, 2000);
- place value as the key to teaching computation with our base-ten numerals (Ashlock, 2009);
- the understanding of zero (Fosnot & Dolk, 2001);
- the whole numeral represents a whole quantity, the sum of the parts, and the individual digits represent a part of the whole collection: a tens part and a ones part (Ross, 1986);
- the ability to decompose and compose numbers in a variety of ways and recognition of the relationship among quantities (NCTM, 2010); and
- canonical representations that have no more than nine objects in any position and noncanonical representations that allow more than nine and was necessary for multidigit algorithms should both be utilized (Ross, 1986).

Additionally, the teachers applied their understanding of place value as they examined the common student misconceptions of addition and subtraction as described by Robert Ashlock (2009) in *Error Patterns in Computation: Using Error Patterns to Help Each Student Learn.* The professional development also included discussions and activities pertaining to using flexible and mental strategies to solve addition and subtraction problems. A copy of the PowerPoint® used to facilitate the two-day professional development activities is included in Appendix B.

Instructional Coaching

Alone, professional developments were probably not enough to change instructional practice (Stein & Nelson, 2003) as professional developments often used a one-size-fits-all approach. Instructional coaching provided a means for differentiation to meet the individual needs of the four participating teachers and helped facilitate the development of the' pedagogical content knowledge of place value. Instructional coaching opportunities for this study included

- modeling new practices for the teachers (Knight, 2007);
- using student work to scaffold teachers' understandings (Peterson, Taylor, Burnham, & Schock, 2009);
- facilitating conversations to discuss observation data (Knight, 2007);
- assisting teachers in developing a content specific lesson (West & Staub, 2003);
- asking questions to prompt conversations instead of telling teachers observation data (Peterson et al., 2009).

Methods in Data Analysis

Quantitative Methods

Learning Mathematics for Teaching

Initially, the LMT mean scores of the individual participants and the population were determined. This analysis allowed for the selection of the study's four participants. Next, it was necessary to analyze the results of the individual test items related to place value and operations. Due to the limited size of this study, the item response theory (IRT) scores provided by the LMT assessment were not valid for this data analysis. Therefore, other methods of data analysis provided insight into the teachers' understandings. In order to gain information as to which concepts the teachers struggled with or had a good understanding of, in regards to place value, determining the mean score of each LMT question was necessary. Analysis of each question provided a big picture of the teachers' understanding of place value concepts or pedagogy. Due to the large number of items, 19 out of 27, in which the teachers showed deficient knowledge, those questions with less than 70% answering correctly—questions pertaining to whole number concepts and the operations of addition and subtraction, which are critical to second grade mathematics instruction—were identified. Table 3 lists the percentage of the population that responded correctly, whether the participants' answer choices for each of these place value and addition and subtraction second-grade-specific questions were correct or incorrect, and the percentage of the four participants who correctly answered each question. Participants who responded to the question correctly received a "1" and participants who answered incorrectly receiving a "0." Additionally, each

participant's percentage of correct answers for these specific questions was figured. From this data, the pedagogy and concepts covered helped to inform the individual instructional coaching efforts for each teacher.

Table 3 LMT pretest item analysis of questions specific to grade 2 mathematics

Population Percentage	Participant A	Participant B	Participant C	Participant D	Percentage Correct
Place Value Questions					
68%	0	1	1	0	50%
73%	1	1	1	0	75%
36%	1	0	0	0	25%
95%	1	1	1	1	100%
59%	0	1	1	1	75%
Addition or Subtraction Questions					
37%	0	0	0	0	0%
41%	0	1	0	0	25%
31%	1	0	1	0	50%
Participant Percentage	50%	62.5%	62.5%	25%	

Concept/Topic	Participant Needing Support
Decomposing number in a variety of ways	A, B, C, D
Values of base ten blocks	A
Using place value to subtract two-digit numbers flexibly	A, B, C, D
Using place value to subtract three-digit numbers flexibly	A, B, C, D

All four of the participants struggled with the understanding a number could be represented in a variety of ways, such as 46 could be represented concretely or pictorially with 4 tens and 6 ones or 3 tens and 16 ones, and was a focus during the instructional coaching opportunities. The teachers and I discussed how an understanding of place value allowed students to flexibly add or subtract without having to memorize a systematic procedure. For example, subtraction with regrouping provided the understanding that if one 10 was taken from the tens place, then 10 had to be added to the ones place. This coaching collaboration also included developing the knowledge of the value of the digits so students know that when adding 29 + 23, they could add 20 + 20 to get 40, 9 + 3 to get 12, and 40 + 12 to get 52 without ever having to follow the traditional procedures for solving addition algorithms.

At the end of the instructional coaching opportunities, the four participants completed a LMT posttest, which allowed for pretest and posttest comparisons. Table 4 shows the four participants' pretest and posttest results for the questions most closely related to the pedagogical content knowledge necessary for teaching second-grade mathematics. Once again, participants receiving a "1" responded to the question correctly, and participants receiving a "0" answered incorrectly. The table also compares the participants' overall percentage of correct answers on the pretest and posttest for the questions pertaining to second-grade pedagogy and content.

Table 4 LMT pretest and posttest item analysis of questions specific to grade 2 mathematics

Pretest Percentage Correct	Posttest Percentage Correct	Participant A	Participant B	Participant C	Participant D
Place Value Questions					
50%	100%	1	1	1	1
75%	100%	1	1	1	1
25%	100%	1	1	1	1
100%	100%	1	1	1	1
75%	75%	1	1	1	0
Addition or Subtraction Questions					
0%	25%	1	0	0	0
25%	50%	1	1	0	0
50%	75%	1	1	1	0
Pretest	50%	62.5%	62.5%	25%	
Posttest	100%	87.5%	75%	50%	

Qualitative Methods

Qualitative data analysis followed the suggestions of Creswell (1998), which included preparing and organizing the data, reducing data into themes through the coding process, and creating a representation of the data. Through instructional coaching opportunities, qualitative data was collected from the four instructional coaching sessions. Table 5 shows a summary of the task completed for each instructional coaching session.

Table 5 Instructional coaching sessions

Participant	Session 1	Session 2	Session 3	Session 4
Participant A	Observation: Building three-digit numbers	Observation: Rolling dice to make and compare numbers	Observation: Creating multiple representations with numbers	Planning: What steps to take next in regards to using place value to add and subtract
Participant B	Observation: Make 10	Model Lesson: Creating set of tens and ones	Observation: Building a number line	Planning: What steps to take next in regards to place value
Participant C	Observation: Math warm-up	Observation: 5E multiple representation lesson (Engage, Explore, and Explain)	Observation: 5E multiple representation lesson (Elaborate and Evaluate)	Observation: Using flexible strategies
Participant D	Observation: Building two-digit numbers	Observation: Building Multiple Representations of two-digit numbers	Observation: 5E multiple representation lesson (Explore)	Observation: 5E multiple representation lesson (Evaluate)

During the study, the instructional coaching opportunities included modeling, analyzing student work, developing lessons, content and pedagogy discussions, and classroom observations. Modeling opportunities included teaching an activity to a class of second grade students or modeling how to use manipulatives for a teacher. Classroom

observations provided opportunities to look for evidence of place value content and pedagogical practices. During follow-up conversations, there were opportunities to discuss student work samples and identify misconceptions or for a participant to ask questions particular to her situation. Key questions such as "How did your observations of your students compare with what you had planned for them to do?" or "What did you observe your students doing well or struggling with during the lesson or activity?" were also included as necessary. Furthermore, instructional coaching included planning with the participants for upcoming place value activities and lessons.

Observation Data Recording Sheet

Observations and reflections were recorded for each classroom observation on an observation data recording sheet. Data from the classroom observations were recorded in the "Observations" column of the recording sheet. After each classroom observation, reflections were recorded in the "Reflections" column. The recorded reflections served as discussion points for the post-observation conversations. The classroom observation data recording sheets from each classroom observation, including the teacher's response to the discussion questions (where applicable), and the anecdotal notes from the model lessons and lesson-planning sessions are located in Appendix C. Table 6 provides an example of the recorded classroom observation and reflection data.

Table 6 Sample classroom observation data recording sheet

Observations	Reflections
Students working in small groups. T: Noticed dice flying through the air so that student group was directed. What are you working on? • S1: We are playing a game and rolling dice to make a number. • S2: I am winning. • T: Remind me how do you play this game? • S2: We each roll a dice and we have to write the numbers down in any order then see who has the biggest number. • S1 rolls a three, S2 rolls a four, and S3 rolls a 6 then S1 recorded 346, S2 recorded 643, and S3 recorded 634.	• Teacher redirected students by asking them questions about the activity. • Did the teacher previously model this activity? • How are students held accountable for work when the teacher is working with other groups? • Students seemed engaged . . . wanting to "play." • Are the groups without the teacher asking questions making connections to the value of the digits?

Semistructured Interview

To gain further insight and to clarify the meaning of the participants' LMT responses, the participants completed a semistructured interview to help determine what to expect from second grade teachers. The semistructured interview considered each participant's individual LMT responses to the questions and provided a means for determining areas to address during future professional development and/or instructional coaching. Figure 2 is a sample recording sheet from one interview with a study participant. A completed semistructured interview for each participant is located in Appendix D.

Figure 2 Sample semistructured interview

Participant: A	Date of Interview: September 2012
• Which questions were the easiest for you? • Which ones were the most difficult?	The question with subtraction was easy because I was able to work it out. The most difficult was question 15 because I do not know how base ten blocks are later used for decimals.
Looking at the questions on the LMT, how did you reach your conclusion? • Why does this work? • What is happening in the regrouping part? • Does the value of the number change when we regroup? Why or why not? • What does the small number represent?	Question 1: We talked a lot about the variety of representations and this understanding helped me answer this question. I did not notice the first time that one of the questions said "tenths." I did not read carefully.
	Question 15: At first I thought the one (units cube) was the whole then I had to think about the fact that it takes 100 of the units to make a hundreds flat or whole.
	Question 10: If you add 10 to ones place, you can add a value of 10 to the other place (tens) to make the numbers friendlier. As long as you do the same to both the tens and ones place, it works. The small number is the 3 tens, or 30.
	Question 13: This problem could be solved in the three different ways. The first time I thought about place value and subtracting the hundreds, then tens, and then ones. It wasn't until the second time to take the test that I saw the student counted up to find the answer in A and that he or she made friendly numbers by adding 4 and 40 to both numbers to make them friendlier.
	Question 25: This one was hard to figure out because we always say you can't take 9 from 7. However, we forget about negative numbers; once I remembered it was simple to say –2 + 20 = 18.
What additional support do you feel you need in regard to place value?	I would like help with differentiating for students who are still struggling to get the basics (counting by 10s, making a number, etc.) I would like more quality activities and model lessons.

The gathered qualitative data and anecdotal records from classroom observations and interviews of the four participants were organized and coded based on common themes. Specifically, to analyze this qualitative data, I read though the classroom observation data forms and anecdotal records with the participants' responses and highlighted any keywords or phrases in which participants may have shared experiences in regards to the understanding or teaching of place value.

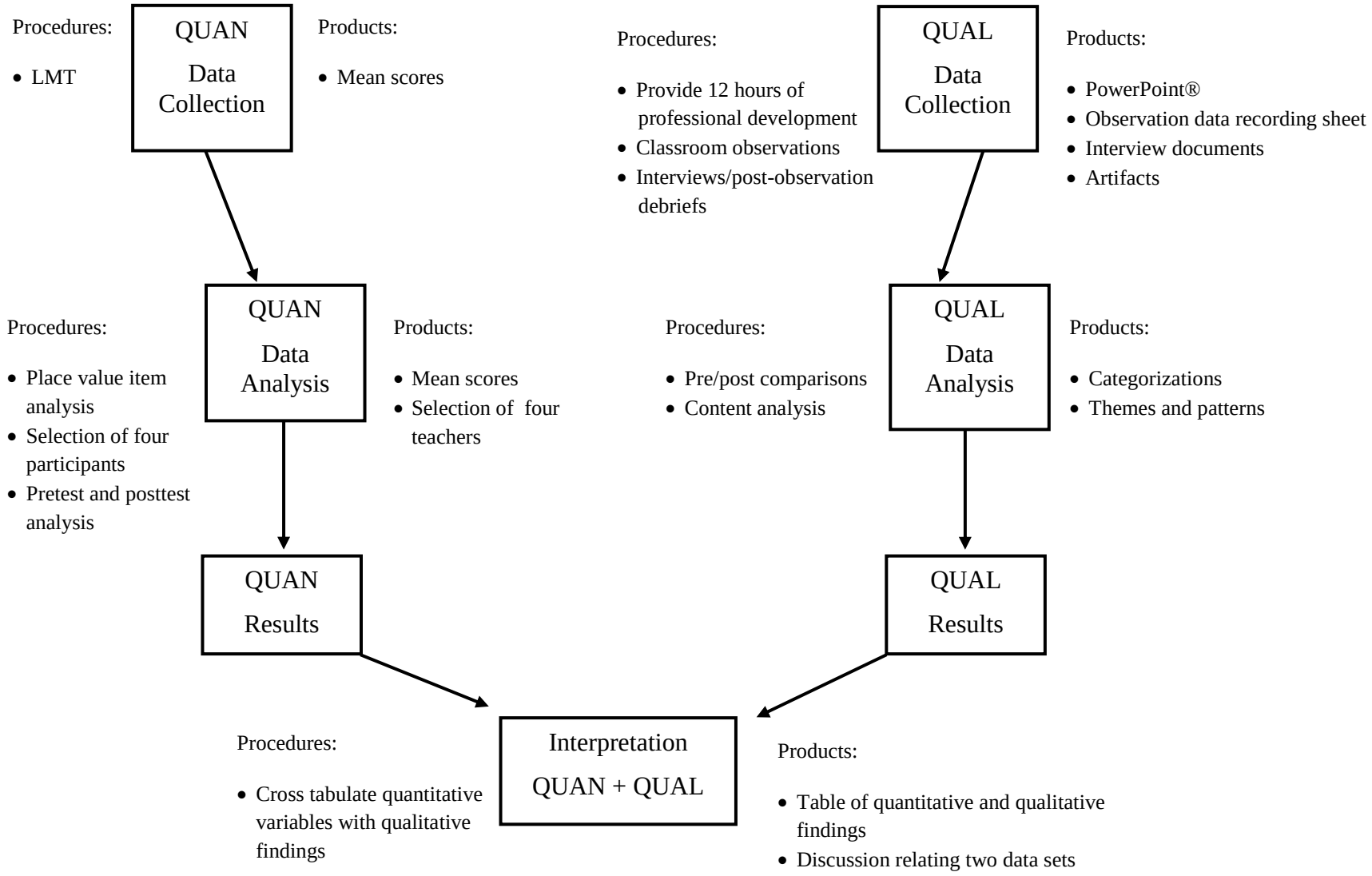
Timeline

The following table shows the sequence of steps I followed to develop, collect, and analyze the necessary qualitative and quantitative data. Table 7 shows the related instruments or protocols related to an activity. The tree diagram in Figure 3 shows the steps specific to data analysis.

Table 7 Research steps to administer activities and instruments related to the solution of the problem

Step	Activity	Dates (2012)	Instrument/ Source	Audience
1	Explain purpose of study and invite participants to attend professional development	May		District teachers
2	Design professional development	July		
3	LMT pretest administration	Aug.	LMT/U of MI	Participants attending PD
4	Provide professional development #1	Aug.		Participants attending PD
5	Provide professional development #2	Aug.		Participants attending PD
6	Analyze test data to select four participants	Aug.	LMT/U of MI	
7	Communicate with four participants and set dates for instructional coaching, observations, and interviews	Aug.		Four second-grade teachers
8	Provide instructional coaching and conduct observations and post-observation interviews	Sept.	Observation forms and anecdotal	Four second-grade teachers
9	LMT posttest administration	Sept.	LMT/U of MI	Four second-grade teachers
10	Conduct semistructured interviews over LMT posttest	Sept.	Appendix B	Four second-grade teachers
11	Analyze quantitative data and qualitative data	Sept.—Oct.	LMT data, observation notes, and interviews	Four second-grade teachers

Figure 3 Steps for data analysis



Theoretical Issues

Reliability

Golafshani (2003) referenced various definitions of reliability. Collectively, these definitions described reliability as a measurement that remained the same or stable over time or as measurements that produced results that were repeatable or replicable. In other words, reliability was the consistency of the measurement. If an instrument was used in the same way, under the same conditions, with the same subjects, and was free from random errors of measurement, then the measurements should be consistent. Reliability for all forms of the LMT was determined through pilot studies from 2001to 2003, with kindergarten through eighth-grade teachers at .70 or above, which shows a moderate effect for studies with 60 or more individuals (Hill et al., 2007). Given the small population size of this study, consideration to the LMT's reliability for populations of 60 or more individuals was necessary when comparing results.

Validity

In quantitative and qualitative research, validity had different meanings; therefore, while conducting mixed methods research, it was necessary to consider both meanings. In qualitative research, the concept of validity must be considered in light of reliability to produce research results that have "credibility, transferability, and trustworthiness" (Golafshani, 2003). However, in quantitative research, validity was determined by whether the research study had measured what it intended to measure and whether or not the means of the measurement were accurate (Golafshani, 2003).

Additionally, there were various kinds of validity; the two types of validity influencing this study are internal validity and external validity. Internal validity helped determine if there was a causal relationship between "A" and "B" (Shadish, Cook, & Campbell, 2002). Internal validity factors, such as history, may influence the study, so as a researcher, I determined whether participants attending additional professional developments resulted in their increased content knowledge. In order to address this concern, I asked the participants to document all professional development opportunities outside of this research study. External validity considered if we could apply the same treatments from the study and obtain the same results somewhere else or whether the results are only applicable to the sample or population of the present study (Shadish et al., 2002). In order to address external validity, I reported relevant information such as gender, ethnicity, and personological variables such as the participants' years of experience and their previous mathematics trainings. This information might allow other researchers to conduct future studies in order to help find solutions to increase teachers' pedagogical content knowledge.

Both of the issues, reliability and validity, were addressed in my research study. By determining and addressing the factors that influence reliability and validity, the findings of second-grade teachers' pedagogical content knowledge of place value could be better understood. However, efforts to reduce issues negatively affecting reliability and validity must be incorporated with the mixed method approach and with the triangulation of data. Together, all of these efforts provided reliable and trustworthy understandings of the concepts presented in my record of study.

Ethical Concerns

Anytime human subjects are involved, a researcher must consider ethical concerns. In this study, I addressed ethical concerns in the following manner: participants received written and verbal explanations of the research project and were informed of their right to not participate as well as their ability to withdraw from the research study at any given point (Appendix A). I explained the confidentiality of the participants' scores, discussions, observations, etc., to the participants. During the process, I provided the participants with a code, used only by me, to connect the data to the participating teacher. When not in use, the codes and the data collected were stored in a locked place in order to assure confidentiality. The participants received assurance that compiled audio recordings would not be released to anyone outside of the research study and no identifiable teacher names would be transcribed into reports. Every effort was taken to maximize the benefits for the participants and minimize the risks.

Results

These findings are based upon the quantitative and qualitative methods previously described. The findings describe what you might expect to see in the classroom teaching of place value from a teacher attending 12 hours of professional development followed up with instructional coaching.

Quantitative Findings

The analysis of the pre- and post-LMT data showed an improvement in every participant's percentage answered correctly. Participant A increased her score by 50%, Participant B increased her score by 25%, Participant C increased her score by 12.5%,

and Participant D increased her score by 25%. Overall, the selected place value questions were answered successfully by the participants: only Participant D missed one question. A misunderstanding about how to apply understandings of place value to solve problems in a manner other than the traditional algorithm still exists. The following figures, Figure 4, Figure 5, and Figure 6, show sample subtraction problems in which the participants struggled to explain why or how the process worked and whether or not it would always work given a different set of numbers. To maintain the reliability of the LMT, the exact questions are not provided.

Figure 4 Adding ten to each place

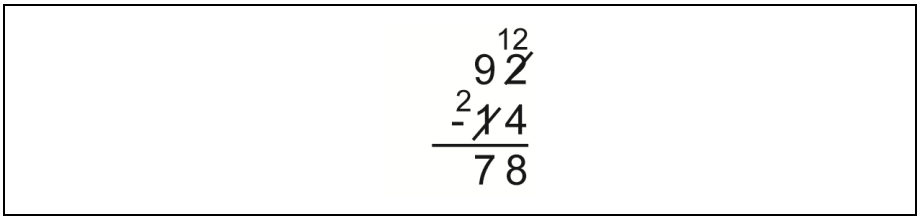


Figure 5 Finding friendly numbers

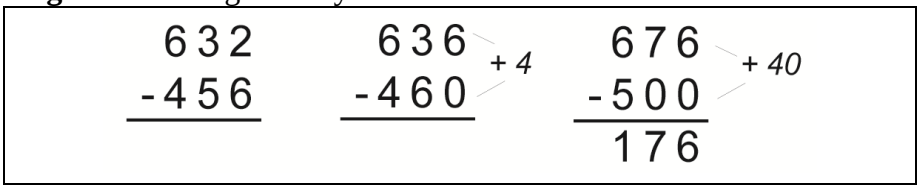


Figure 6 Using negative numbers

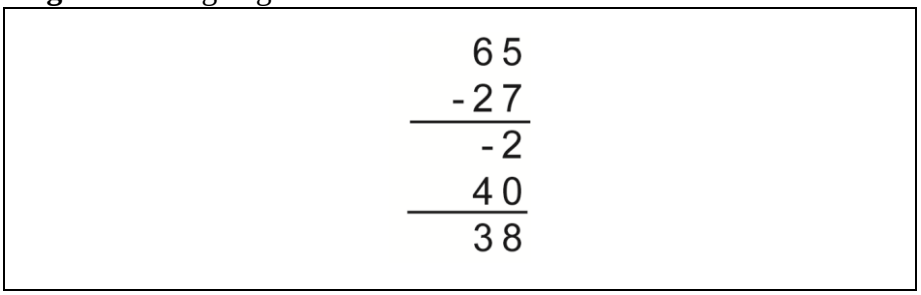


Figure 4 involved adding a 10 to both the tens and ones place. Participant A was the only one to successfully answer this question. Figure 5 included adding 4 to both of the numbers then adding 40 to the new numbers so that it was easy to subtract. Participant A and Participant B answered this type of problem correctly. Figure 6 involved the understanding of negative numbers; while negative integers are not introduced in second grade, the teachers should have an understanding of how and why this solution method is feasible. Participants A, B, and C all answered the question correctly. These three problems challenged teachers to use their understanding of place value in a flexible manner.

Qualitative Findings

Participant A

Instructional opportunities for Participant A included three classroom observations followed by a discussion with the participant and one lesson-planning session. Table 8 provides a summary of the lesson observation, lesson planning, and post-conference for each of the four instructional coaching sessions completed for Participant A.

Table 8 Summary of Participant A's instructional coaching sessions

Session 1	Session 2	Session 3	Session 4
Observation: Building three-digit numbers	Observation: Rolling dice to make and compare numbers	Observation: Creating multiple representations with numbers	Planning: What steps to take next in regard to using place value to add and subtract
Coaching: Build numbers with a variety of concrete representations • Unifix© Cubes for students struggling to see the "long" as a representation of 10 • Bundling straws • Unifix© Cubes	Coaching: Some students struggled with ordering the digits to create the greatest number. • Build numbers first using the H-T-O Mat and base ten blocks • Add a recording sheet for the values of each digit	Coaching: Some students are still saying 2 + 4 + 1 • Record each place's value ___H___T___O • Provide a calculator to let the students add the values • Continue skip-counting and counting on conversations	Coaching: Planning for incorporating addition and subtraction concepts for struggling students, on-level, and advanced students

Pedagogical data showed that Participant A interacted with her students through small-group instruction. During small-group instruction, Participant A grouped the students according to their understandings of place value concepts. Each lesson observed contained one of the instructional activities discussed during the summer professional development. Observation 1 included building numbers with base ten blocks on the hundreds, tens, and ones mat. Observation 2 included rolling the dice to generate numbers, then using place value understandings about numbers to create the largest number possible. Observation 3 included using a recording sheet from the professional development, which allowed students to connect multiple representations of base ten

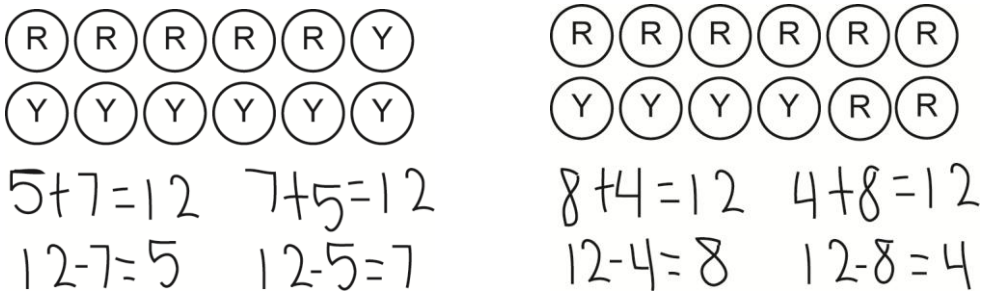
blocks to the corresponding pictorial models. During the observations, I noticed that, each time, the students had the opportunity to share their ideas and explain ideas, such as why or how a base ten model represented a given number. In addition, Participant A implemented the use of questioning that was modeled and emphasized during the professional development and instructional coaching. The questions included "How many tens?", "What is the value of the tens?", and "How could you use skip-counting and counting-on to find the total value of the base ten blocks?"

A discussion followed each of the three classroom observations. During those discussions, Participant A asked many pedagogical questions she felt would help her improve her mathematics instruction. Several discussions involved modifying instruction for students struggling with place value concepts. For example, "How do you scaffold instruction so that it is accessible to all students?" or "How and when do you move the students from concrete representations to pictorial representations?" Being able to differentiate appropriately for her students was what Participant A felt she needed further support on through professional development or instructional coaching. In response, to Participant A's needs, I provided additional resources and suggestions to help her differentiate the lesson just observed for struggling students. The ideas included explaining how to use a variety of manipulatives such as Unifix© Cubes, which help students bundle 10 objects to together to build one "train" before they accept the fact that one "long" represents a quantity of 1 ten or 10 ones. Participant A was provided with additional activities and recording sheets to prompt the students to record the number of

hundreds, the number of tens, and the number of ones and link the digit to its value based on its place.

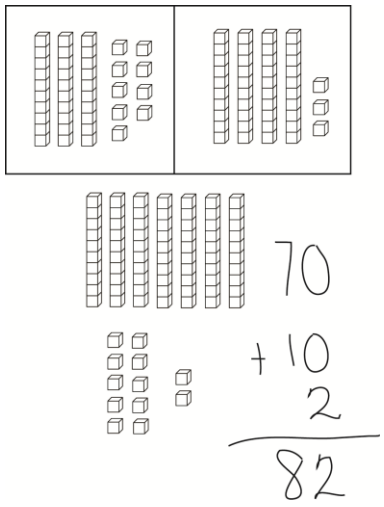
During one lesson-planning session, Participant A and I collaborated to plan upcoming addition and subtraction activities, which built upon place value concepts. For struggling students, I used a set of two-colored counters to model for Participant A how to decompose and compose small quantities and record the corresponding addition and subtraction number sentences (Figure 7).

Figure 7 Example of composing and decomposing 12



For students developing a firm concept of place value, we discussed prompting students to use base ten blocks to represent and build two different numbers and then allowing them to explore a variety of ways to combine the sets in order to determine the total number, such as the example shown in Figure 8.

Figure 8 Using base ten blocks to add flexibly



This concept was new for Participant A because in the past, addition and subtraction have been taught through the traditional algorithm methods of adding the units, regrouping 10 units and trading them for one long if necessary, and then adding up the longs. In line with the literature review findings, the hope was for students to use their understanding of place value and be able to add the set of base ten blocks flexibly. Next, we discussed the importance of prompting students to share their solution strategies by describing the value of the base ten blocks or digits.

Participant B

Instructional opportunities for Participant B included two classroom observations with follow-up discussions, one model lesson, and one planning session. Table 9 provides a summary of the lesson observation, model lesson, lesson planning, and post-conference for each of the four instructional coaching sessions completed for Participant B.

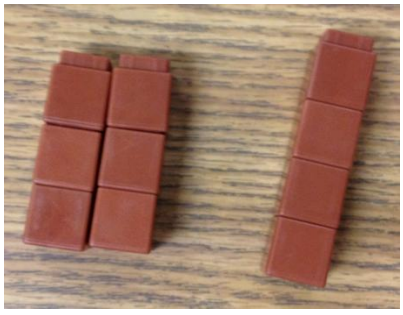
Table 9 Summary of Participant B's Instructional Coaching Sessions

Session 1	Session 2	Session 3	Session 4
Observation (and modeling): Make 10 Coaching: Co-taught the lesson • Provide model questions • Allow students to mentally compose and decompose numbers or use objects as needed	Model Lesson: Creating sets of tens and ones Coaching: Explained the value and purpose of allowing students to create sets of tens and ones • Need to bundle sets before using prebundled sets such as the base ten blocks • Use Unifix© Cubes and mini ten-frame mats to help bundle groups of tens	Observation: Building a number line Coaching: Continuing to represent 2-digit numbers Role of the number line in developing place value and number sense	Planning: What steps to take next in regards to place value Coaching: Planning for developing the concept of place value

Based on Participant B's individual instructional needs, there were only two classroom observations. Before Observation 1, Participant B e-mailed me and asked for help with getting the students to explain their thinking. I gave thought to my response and was prepared to discuss these ideas during a post-observation conference. During this observation, students began to mentally compose and decompose the number 10, which represented the teacher's implementation of understandings from that professional development: numerical fluency and number sense, a requisite for place value understanding, and addition and subtraction could be taught simultaneously. At the

beginning of the lesson, Participant B stopped and asked if it would be appropriate to use concrete modeling for the activity since some students struggled to compose and decompose the number. We distributed Unifix© Cubes and asked students to compose and decompose small numbers in a variety of ways, which was similar to an activity from our professional development. She allowed the students to explore the possible combinations; then she asked the students to tell her the addition number sentence that could be used to represent their model. Participant B asked for types of scaffolding questions that might help struggling students and extend the concept for those students who needed enrichment. For example, in Figure 9, one student decomposed the Unifix© Cubes into two parts and described the model as 6 + 4. I provided support by prompting students to describe this representation in a different way, such as 2 + 2 + 4.

Figure 9 Using Unifix© Cubes to compose and decompose numbers into two parts



Furthermore, I challenged students to compose and decompose the number 10 into more than two parts. Figure 10 shows how one student decomposed the number 10 into four parts: 1 + 3 + 3 + 3.

Figure 10 Using Unifix© Cubes to compose and decompose numbers into more than two parts

After this observation, Participant B and I discussed how addition and subtraction number sentences could describe the composing and decomposing of the number 10. For Observation 2, I observed a whole-group lesson in which the students worked to create a number line for the numbers 0 to 20. During the professional development, activities included the use of an open number line to develop an understanding of place value and number sense. Participant B also practiced pedagogical strategies from our training such as allowing students to change their predictions or answers as they gained further information.

A post-observation conference followed each classroom observation. After Observation 1, we discussed how composing and decomposing a number, mentally or with objects, should prompt students to record addition and subtraction number sentences. Additionally, we discussed students describing a variety of ways to make a number. Participant B expressed concern about teaching the pedagogy and conceptual development of the mathematics concept. For example, she asked, “Is this a worthwhile activity?” and she stated, “I should have started with counters.” Observation 2

discussions included a conversation about place value and the concept of number sense and the role of a number line in the development of these concepts. Participant B was the only teacher from her campus that attended the summer professional development, which left Participant B, new to second grade, with the task of trying to synthesize ideas she gained about place value with her team's lesson plans and instruction. To make sure I did not provide any suggestions contrary to the district's scope and sequence, I consulted Dr. Johnson before the follow-up instructional coaching planning session.

The two instructional coaching opportunities included a request to model a lesson that extended the ideas and concepts presented during the 12 hours of professional development and a planning session to continue the post-observation discussions from Observation 2. During the model lesson, I demonstrated the concept of bundling groups to create sets of tens and ones. This lesson built to the understanding of what the prebundled base ten blocks models represent. Too often, teachers take for granted that telling a student a "long" represents a 10 makes the student understand the meaning of a group of ten. The planning session provided Participant B with the next steps in continuing place value instruction in her classroom in a manner that developed students' number sense and concept of place value. This lesson-planning session included developing the concept by moving to base ten blocks and connecting the values to the models, creating multiple representations, comparing and ordering the models, comparing and ordering on the number line, and moving to abstract understandings.

After completing the instructional coaching with Participant B, I received the following e-mail, which celebrated the successes of our instructional coaching conversations.

> Oh my gosh! You said they would use the language. A student was explaining her thinking today and said "19 is higher than 15 so it would go here" on the number line. They caught that it would not even fit on a number line 5-15. I commended them for their thinking. Then I mentioned another way to say higher is more. They used less in the conversation later. I was so excited I could hardly contain my excitement! Feeling the direction now! Think I know where we are going for now. Thanks for working through that with me today! And for the books! Some amazing things may happen in this room this year. Thanks again! (personal communication, September 14, 2012).

Participant C

Instructional opportunities for Participant C included four classroom observations with follow-up discussions. Table 10 shows a summary of the lesson observation and post-conference for each of the four instructional coaching sessions completed for Participant C.

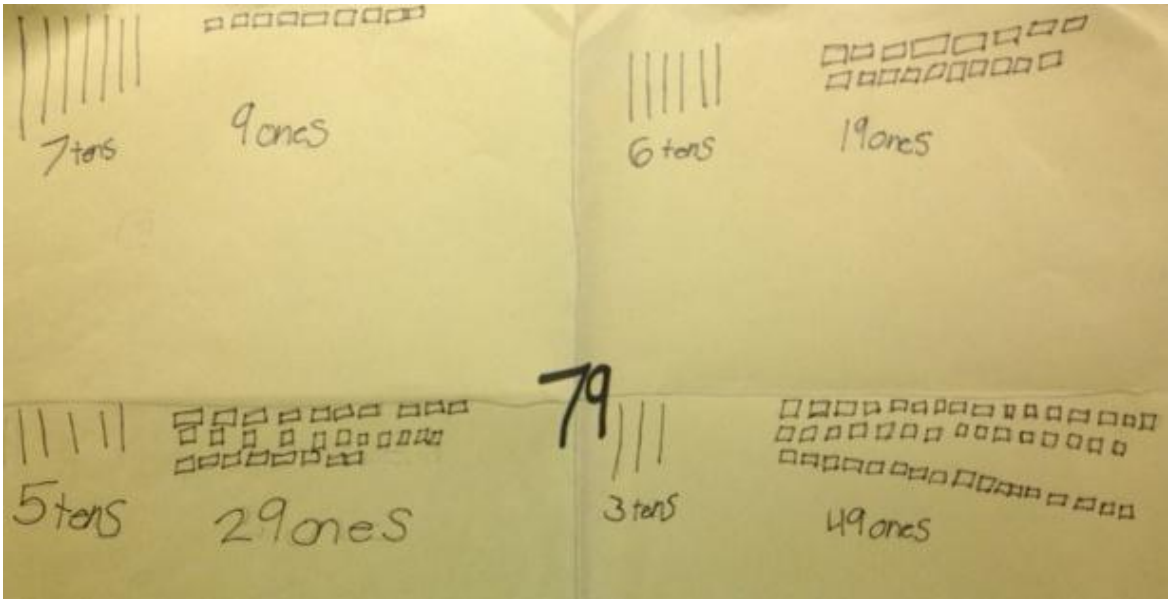
Table 10 Summary of Participant C's instructional coaching sessions

Session 1	Session 2	Session 3	Session 4
Observation: Math warm-up Coaching: • Shared success story from trying ideas presented during the professional development • Continue to encourage students to talk and share strategies and ideas	Observation: 5E Lesson: Multiple Representations of 2-digit numbers Coaching: • Connect models to the values (written and expanded form)	Observation: 5E Lesson: Multiple Representations of 2-digit numbers Coaching: • Recording the multiple representation values • Clarifying the use of expanded notation • Student work	Observation: Using flexible strategies to add and subtract Coaching: • Discussed how this campus plans math instruction

During the classroom observations, Participant C taught the mathematics lessons through whole-group instruction, which included opportunities for students to collaborate with one another. During Observation 1, the students responded to several place value warm-up questions. The questions included "What is another way to write 25?", "What is another way to write 71?", "What is the value 60 + 500 + 8?", What is the value of 10 ones, 5 hundreds, and 0 tens?", and "What is the sum of 4 + 4 + 4?" Students solved these problems individually; then shared their thoughts with their

"elbow partner" to clarify and refine their answers before discussing the answers with the whole group. During Observations 2 and 3, Participant C taught the lesson using the 5E instructional model. Throughout this lesson, the students built upon their understandings of building two-digit numbers and began to develop the understanding that numbers could be represented through a variety of representations. Figure 11 provides an example of the evaluate portion of the lesson, in which the students were required to work together to represent a number in four different ways.

Figure 11 5E lesson—Evaluate



The fourth observation provided the opportunity to observe numerous students from Participant C's class successfully add three-digit numbers flexibly by using place value. For example, given the problem 211 + 303, a student successfully explained that

200 + 300 = 500 and 11+ 3 = 14, so the answer was 514. Another student explained that 200 + 300 was 500 and there was only one 10 so it was 500 +10, which was 510. Next, you had to add 3 + 1 = 4, and then add 500 + 10 + 4 to equal 514. Based on traditional second-grade scope and sequences, this type of thinking would not have occurred in the second-grade mathematics classroom until late in the year, if at all.

Each of the four observations was followed by a post-conference discussion. During the first discussion, Participant C excitedly shared a success story from ideas presented during the professional development. After trying the activities involving composing and decomposing numbers into friendly or manageable numbers, Participant C said, "I am shocked that my students can actually use what they learned about place value in first grade and apply those understandings to solving an addition problem in a flexible manner." She said her next goal was to get the students to more efficiently communicate about mathematics: how they solved a problem or how they knew their answer was correct. After Observation 2, her goal was getting students to understand the importance of recording the value of models in written, word, and numerical form: 56, fifty-six, 5 tens and 6 ones. The Observation 3 post-conference provided opportunities to talk about the student work and the value of students communicating. The last observation provided me insight into how this campus, the only campus in this study not in trouble for failing to meet AYP, plans their mathematics instruction.

Participant D

Instructional opportunities for Participant D included four classroom observations with follow-up discussions. Table 11 shows a summary of the lesson

observation and post-conference for each of the four instructional coaching sessions completed for Participant D.

Table 11 Summary of Participant D's instructional coaching sessions

Session 1	Session 2	Session 3	Session 4
Observation: Building two-digit numbers with objects Coaching: • Connect base ten blocks to the value of the tens, ones, and the total value. • For accountability purposes, students can record the value of the tens, ones, and combined value.	Observation: Building multiple representations of two-digit numbers Coaching: • Offered assistance to provide additional support.	Observation: 5E multiple representation lesson (Explore) Coaching: • Discussed the 5E instructional model.	Observation: 5E multiple representation lesson (Evaluate) Coaching: • Allow students to add values to the pictorial models created for the Evaluate.

The instructional coaching opportunities with Participant D were cordial; yet the teacher did not seem interested in discussing ways to improve instruction. Of the four participants' classrooms, Participant D's classroom was the most teacher centered as compared to the other participants' attempt to be student centered and build a community of learners. For this participant, the role of the instructional coach was to

provide resources and let the teacher use them or come show her how to use them. During the four observations, the students did use base ten blocks to represent numbers. However, there was not a focus on the value of the digits. For example, when asked to represent the number 52 with base ten blocks, most students could model the number, as shown in Figure 12. However, several students in Participant D's class counted out 5 tens and 2 ones and said, "I have 5 and 2."

Figure 12 Model of 52 using base ten blocks

Throughout the four observations, the students showed they were able to build two-digit numbers, but there was little evidence of the students' ability to connect the base ten models to the value they represented. Participant D did not ask students follow-up questions or allow for classroom discussions that may have enhanced the students' understanding of place value. As far as instructional coaching efforts, the post-observation conversations were challenging in that there was a feeling of resistance.

After providing suggestions to Participant D, statements such as “I know” or “I already do that” were often given in response.

Triangulated Findings

Together the quantitative data from the LMT and the semistructured interview provided further insight into each participant’s pedagogical content knowledge. After professional development and instructional coaching, Participant D was the only one to miss a question related to place value. Ironically, Participant D answered this question correctly on the pretest. During the posttest interview, Participant D mistakenly described the unit as the whole instead of the hundreds flat representing the whole.

There were three subtraction problems closely related to second-grade concepts. The first question required the participants to add the value of 10 to the tens and ones place. The only participant to answer this correctly was Participant A. The other participants assumed that the student marked out the wrong digit and added on one 10 instead of removing one 10. The second question included three methods in which the participants had to determine if the solution methods were correct. The solution method that every participant recognized as valid was the one in which the student used place value to flexibly subtract the hundreds, tens, and then ones. Participant A, Participant B, and Participant D were able to recognize counting-up as a valid solution for solving subtraction problems. However, only Participant A recognized that a problem could be solved by adding the same quantity to each number to make the numbers easier to manipulate. The other participants really did not have an explanation as to why the solution method would or would not work.

CHAPTER III

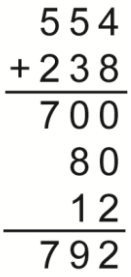
CONCLUSIONS

A thorough analysis of the data from the LMT pretest and posttest, classroom observations, instructional coaching opportunities, and post-LMT discussions helped answer the research question: What can you expect to see in the classroom teaching of place value from second-grade teachers who have attended 12 hours of professional development and have had instructional coaching? This research revealed many consistencies and discrepancies among the four participants.

The LMT pretest and posttest data showed an increase in each participant's understanding of place value and in their ability to apply those understandings to the concepts of addition and subtraction. The participants increased their scores by the following amounts: Participant A, 50%; Participant B, 25%; Participant C, 12.5%; and Participant D, 25%. The percentage gains show that 12 hours of professional development does in fact help increase teachers' pedagogical content knowledge. However, when combined with the qualitative data from the semistructured interviews, further information can be gained. For example, of the questions selected, the only misunderstanding, by Participant D, pertained to the understanding of also being able to use base ten blocks as tools to teach decimal concepts and is not a critical component of second-grade mathematics. As for the other incorrect responses, the participants lacked an understanding of how to add or subtract using compensation or by using negative numbers. The data revealed that all of the participants recognized the flexible method for

addition and subtraction most directly correlated to second-grade understandings (Figure 13). This supports the fact the participants gained an understanding of concepts presented after professional developments and instructional coaching.

Figure 13 Adding flexibly



Additionally, the analysis of the qualitative data revealed several themes common to all of the participants in regards to their experiences with the professional development or instructional coaching: classroom implementation of concepts, specific mathematics pedagogy, and the affective value of instructional coaching. Observation data showed that every participant implemented activities and concepts from the two-day professional development in her classroom. Although each teacher was following the same district scope and sequence, the mathematics lessons observed varied. However, from the various key understandings from the professional development, all of the participants embraced the idea of multiple representations of a number because they felt that in the past they had solely focused on representing 56 as 5 tens and 6 ones. Each participant deemed this concept as critical to second grade because their students have always struggled with addition and subtraction involving regrouping.

During classroom observations, every participant asked her students to represent a number using multiple representations. The need for understanding multiple representations is critical to the understanding of the standard addition and subtraction algorithms requiring regrouping. Participant A built, composed, and decomposed three-digit numbers; Participant B composed and decomposed numbers to 20; and Participant C and Participant D found multiple representations of two-digit numbers. However, all but one participant, Participant A, were so focused on the multiple representations of a number that they forgot to talk about the value of numbers being represented such as 53 is 5 tens and 3 ones, or 50 + 3; or 4 tens and 13 ones, or 40 + 13. However, Participant D became so concerned with listing all of the possible representations of a number, it became almost counterintuitive to the concept of developing students' flexible thinking or number sense.

Furthermore, classroom observations revealed that Participant B and Participant C had begun to develop the concepts of addition and subtraction through the understanding of place value, but in different ways. Participant B worked to develop numbers to 20; however, Participant C went beyond the scope and sequence and allowed the students to add and subtract three-digit numbers based on their understanding of place value, as shown in Figure 13. While Participant C missed two questions about flexible strategies on the LMT, her students showed that beginning second-grade students were capable of solving problems using an understanding of place value. After sharing information about Participant C's success, Participant A wanted to develop plans for teaching addition and subtraction in conjunction with place value.

Next, all of the participants used concrete objects (Unifix© Cubes or base ten blocks) to help develop number or place value concepts. What differed among the participants was the discussion about the value of the students' models. Participant A and Participant B asked students to explain the value of the digit in the hundreds, tens, or ones place, then asked students about the combine value of all three places. During the observations, Participant C did not ask students about the values of their base ten blocks yet, some of her students were able to use the value of digits to solve addition problems flexibly. Participant D's students appeared to build base ten models by viewing the digit in the tens and ones place to tell them how many tens or ones were needed. For example, when shown the number 52 students would count out 5 tens and 2 ones without considering the value of the tens as 50 and the value of the ones as 2. Additionally, the teachers all seemed to understand the conceptual development process of moving from concrete representations to pictorial representations and then finally to abstract representations.

During the professional development, we discussed, at length, the need for students to have mathematical conversations with one another. In Participant A's, Participant B's, and Participant C's classrooms, the students worked cooperatively to develop an understanding of the place value concepts in a community- type environment. In Participant D's classroom, the students worked mostly independently during each observation. Additionally, Participant C and Participant D used the 5E instructional model, which included teaching the concept through the Engage, Explore, Explain, Elaborate, and Evaluate phases.

The data collected from the post-observation discussion revealed that instructional coaching provided a way to support the individual needs of each participant and the unique needs of the students in their classrooms. The participants' varied needs included finding ways to improve the lesson through adding recording sheets, by discussing differentiation, by varying the concrete objects, and by discussing ways to plan for future instruction. For future instructional coaching opportunities, the participants felt it would be valuable to have a coach who came in and demonstrated a model lesson, a day-to-day curriculum, and to help differentiate for all of the various ability levels in the classroom.

Together, the quantitative and qualitative data showed that a teacher receiving 12 hours of professional development and some instructional coaching has a basic understanding of place value. The participants were able to use good pedagogy to get the students to represent a number, including representing a number in a variety of ways; however, there is a need to place even more emphasis on the value of the digits or the value the model represents. Based on the LMT data and the semistructured interviews, it was evident that three out of the four participants struggled with applying their understanding to the concepts of addition and subtraction. One source of this problem may be the participants' lack of experience with working flexibly with numbers. Traditional mathematics teachers taught procedures for addition and subtraction without explaining why or how the procedures worked. In order to answer the question of how and why the procedures for addition and subtraction work, one must have a deep understanding of place value. Additionally, it appeared that when provided

classroom-ready activities during a professional development, resources were implemented. Instructional coaching seemed to provide the opportunity for the participants to gain further understanding as needed. As well, the language and questioning strategies modeled during the professional development were also implemented into the classroom.

Furthermore, based on the record of study findings, the district studied needs to develop a sustainability plan, which may include events such as additional professional development and additional instructional coaching to support the teachers' continuous improvement of their pedagogical content knowledge. Due to the district's implementation of the new TEKS, all of the participants expressed uncertainty about the progression of place value and addition and subtraction. Specific needs stated by the participants included

- Where and how do they begin?
- What do you do if students are not successful?
- What do you do next?
- How do you transition to addition and subtraction?

Based on this study, as district leaders plan and provide materials for professional development, it may be helpful to sequence the new activities to the district's scope and sequence or curriculum documents.

Future Studies

This study showed how teachers respond to opportunities given to increase their pedagogical content knowledge. Teachers need to know more than just how to do

procedural skills; they must understand mathematics concepts. Based on this study's findings, future research with a larger sample size, a control group, and a treatment group are needed to demonstrate whether the teachers' pedagogical content knowledge increased because of the professional development and instructional coaching or because of some other factors. However, based on these positive results, there is evidence that even 12 hours of professional development followed by instructional coaching may improve second-grade teachers' pedagogical content knowledge.

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APPENDIX A

APPENDIX A

PARTICIPANT CONSENT FORM

Project Title: Developing Second Grade Teachers' Pedagogical Content Knowledge of Place Value

You are invited to take part in a research study being conducted by Stefani Kulhanek, a researcher from Texas A&M University. **The information in this form is provided to help you decide whether or not to take part. If you decide to take part in the study, you will be asked to sign this consent form. If you decide you do not want to participate, there will be no penalty to you, and you will not lose any benefits you normally would have.**

Why Is This Study Being Done?

The purpose of this study is to determine whether professional development and instructional coaching are effective solutions for increasing teachers' pedagogical content knowledge of place value.

Why Am I Being Asked To Be In This Study?

You are being asked to be in this study because you are currently a second grade classroom teacher in this school district.

How Many People Will Be Asked To Be In This Study?

The approximately 30 participants who attend 12 hours of the professional development will be invited to become a part of this study.

What Are The Alternatives To Being In This study?

The alternative to being in the study is not to participate.

What Will I Be Asked To Do In This Study?

You will be asked to complete a pretest and posttest using the "Survey of Elementary Teachers of Mathematics: Number Concepts and Operations Study." You will be asked to attend 12 hours of professional development in the summer, respond to interview questions, and allow the researcher to provide implementation support through instructional coaching. Your participation in this study may last up to 6 months.

If you leave the study early, you may be asked to complete the following activities: Posttest "Survey of Elementary Teachers of Mathematics: Number Concepts and Operations Study."

Will Photos, Video, Or Audio Recordings Be Made Of Me During The Study?

The researchers will make an audio recording of the teacher during the study so that that data can be gathered about participants' pedagogical content knowledge only if you give your permission to do so. Indicate your decision below by initialing in the space provided.

_____ I give my permission for audio recordings to be made of me during my participation in this research study.

_____ I do not give my permission for photographs/audio/video recordings to be made of me during my participation in this research study.

Are There Any Risks To Me?

The things that you will be doing are no more risks than you would come across in everyday life.

Will There Be Any Costs To Me?

Aside from your time, there are no costs for taking part in the study.

Will I Be Paid To Be In This Study?

You will not be paid for being in this study.

Will Information From This Study Be Kept Private?

The records of this study will be kept private. No identifiers linking you to this study will be included in any sort of report that might be published. Research records will be stored securely and only Stefani Kulhanek will have access to the records.

Information about you will be stored in a locked file cabinet and computer files protected with a password. This consent form will be filed securely in an official area.

People who have access to your information include the principal investigator and research study personnel. Representatives of regulatory agencies such as the Office of Human Research Protections (OHRP) and entities such as the Texas A&M University Human Subjects Protection Program may access your records to make sure the study is being run correctly and that information is collected properly.

We May Be Legally Obligated To Disclose Information Under The Texas Public Information Act. Information about you and related to this study will be kept confidential to the extent permitted or required by law. The Texas Public Information Act provides a mechanism for the public to request public information in Texas A&M University's possession, which may include information about you and/or information related to this study. If Texas A&M University receives a request for public information relating to this study, the university will seek to withhold information about you and/or this study to the extent such information may be considered confidential by law and to the extent legally permitted and authorized by the Texas Attorney General's Office to do so.

Who May I Contact For More Information?

You may contact the principal investigator, Dr. Dianne Goldsby, to tell her about a concern or complaint about this research at (979) 845-8384 or dgoldsby@tamu.edu. You may also contact the protocol director, Stefani Kulhanek at 281-356-2788 or paymak98@tamu.edu.

For questions about your rights as a research participant or if you have questions, complaints, or concerns about the research, you may call the Texas A&M University Human Subjects Protection Program office at (979) 458-4067 or irb@tamu.edu.

What If I Change My Mind About Participating?

This research is voluntary and you have the choice whether or not to be in this research study. You may decide to not begin or to stop participating at any time. If you choose not to be in this study or stop being in the study, there will be no effect on your employment or teacher evaluation. Any new information discovered about the research will be provided to you. This information could affect your willingness to continue your participation.

STATEMENT OF CONSENT

I agree to be in this study and know that I am not giving up any legal rights by signing this form. The procedures, risks, and benefits have been explained to me, and my questions have been answered. I know that new information about this research study will be provided to me as it becomes available and that the researcher will tell me if I must be removed from the study. I can ask more questions if I want. A copy of this entire consent form will be given to me.

_____	_____
Participant's Signature	Date
_____	_____
Printed Name	Date

INVESTIGATOR'S AFFIDAVIT:

Either I have or my agent has carefully explained to the participant the nature of the above project. I hereby certify that to the best of my knowledge the person who signed this consent form was informed of the nature, demands, benefits, and risks involved in his/her participation.

_____	_____
Signature of Presenter	Date
_____	_____
Printed Name	Date

APPENDIX B

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Developing Number and Place Value Concepts

Stefani Kulhanek

Welcome & Introductions

- Welcome
- Facilitator Introductions
- Table Introductions
- Purpose of Training: Doctoral Study

Getting Started

- Create a tri-fold name tent
- Print your first name on one side

Goals

- Explore instruction that supports numerical fluency
- Explore instruction that supports development of place value concepts
- Explore instruction that encourages students to apply their understandings of place value to the operations of addition and subtraction

Record of Study

- Research suggests teacher quality is a significant factor in predicting student achievement (Akiba, LeTendre, & Scribner, 2007; Hill et al., 2005).
- Conceptual understandings refers to an individual's ability to integrate mathematical ideas, represent mathematics in different ways, or use the knowledge of mathematics in a variety of situations (National Research Council 2001).

Record of Study

- Consent forms
- Assessment and questionnaire
- Reflection Sheets

Why take another look at place value, addition, and subtraction?

- Research supports the critical importance of a strong foundation in recognizing part-whole relationships and place value concepts as groundwork for other mathematical understandings (Fischer, 1990; Ross, 1989).

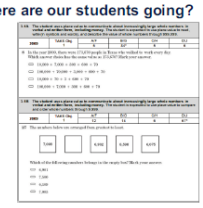
Why take another look at place value, addition, and subtraction?

- Place value, addition and subtraction are foundations of numeracy. Research demonstrates that poor numeracy is more detrimental than low levels of literacy in obtaining jobs and earning promotions (Brynner & Parsons, 1997).

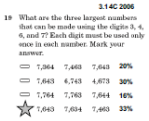
Where are our students going?

- Grades 3-5 TAKS™ Data
  - What does the data tell us about our students' understanding of place value concepts?
  - How do these data relate to YOUR students?

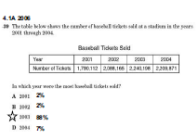
Where are our students going?



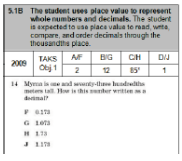
Where are our students going?



Where are our students going?



Where are our students going?



Place Value Partners

Reflect

- How do you define place value?
- How might place value concepts be used to solve 76 x 89 ?

Where are our students going?

- Work with your partner to solve the problem 76 x 89 in three different ways.
- How might place value concepts be used to solve this problem? (76x100)-(76x10)-(76x1)

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Where are our students going?

As students work with numbers, they gradually develop flexibility in thinking about numbers, which is a hallmark of number sense.

NCTM Principles and Standards (2000) p. 80

Reflect

- What is numerical fluency?

Students should develop one-to-one correspondence regardless of the kind of objects in the set and the order in which they are counted.

1:1 Correspondence

- The Counting Circle

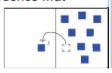
Components of Numerical Fluency

- How are these components similar to and different than your thoughts?

1:1 Correspondence

1:1 Correspondence

- How many cartons of milk do I need for the 18 students present in our class today?
- Counter of the Day
- One-to-One Correspondence Mat
- Counting tracking mat



Inclusion of Set
Cardinality
Conservation

Students should recognize that the last number said indicates the total number of objects, which is referred to as cardinality and the number of the set stays the same regardless of the arrangement, which is the idea of conservation.

Numbers 0-5

- The Number Game
  - Show number in baggie
  - Put cubes on fingers
  - Put cubes back
  - Jump

  - Clap

Numbers 0-10

- Create a set of counters to represent the number 9.
- Organize your set of counters on the Five Frame.
- How could you describe your set of counters?
- Organize your set of counters on the Ten Frame.

Numbers 0-10

- Choose 1 ten frame card.
- Create a set of counters on the Ten Frame that represents the same amount as your ten frame card.
  - What number does the Ten Frame show?
  - How could you use a Number Card to represent that amount?

Numbers 0-5

- Organize 4 counters on the Five Frame.

- What questions could we ask students about their arrangement?
- How does organizing counters on the Five Frame facilitate understanding of number?

Numbers 0-5

- Create a set of counters to represent the number 3 on your Five Frame.
- Represent each counter with a dot sticker on the recording sheet.
- Repeat the process for the remaining numbers.
- How could students use the dot sticker Five Frames to facilitate future learning?

Numbers 0-20

- Create a set of counters to represent the number 12.
- Organize your set of counters on the Ten Frame.
- How could you describe your set of counters?
- Organize your set of counters on the Double Ten Frame.
- How might this help student "count-on"?

Counting-On

- Cups and Counters
- Counting-On Cards

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

- In kindergarten students begin to count forward and backward with or without objects.
- As students become more proficient, through the understanding of counting patterns, students are able to count and forward and backward beginning with any given number.

Reflect

- What is the value of using Five Frames, Ten Frames, and Double Ten Frames to develop students' sense of number?

Subitizing

- Dot plates/cards
- Domino flash/dice roll

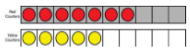
Subitizing

- Quick Images

Subitizing

Students should begin to instantly recognize the quantity of a small group of objects in organized and random arrangements.

More Than, Less Than, Equal To



- Before students have an understanding of number, many understand that there is a relationship between quantities.
  - For example, students begin to compare quantities without numerical values such as comparing the length of two pieces of string.

Comparing Numbers

- Dot comparisons
- More, Same, Less Mats

Part/Part/Whole

- Composing and decomposing sets is an important concept that helps support the students' understanding that 10 ones can be bundled or regrouped into 1 ten, 10 tens can be bundled or regrouped into 1 hundred, and 10 hundreds can be bundled or regrouped into 1 thousand.
- This is essential to students' developing an understanding of the base ten system. This understanding helps students recognize that numbers may be represented in more than one way.

Unitizing

Reflection: Part/Part/Whole

- What does it mean to compose and decompose numbers?

Part/Part/Whole

- Part/Part/Whole Mat
- What Part is Missing?
- Make 10!

Reflection: Unitizing

- What is unitizing? How is it related to the development of place value concepts?
- Understanding that ten can be represented and thought of as one group of 10 or 10 individual units.

- How do the activities we have explored align to the Texas Essential Knowledge and Skills (TEKS)?

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Game Plan

- What should developing number concepts look like in my classroom?

Understanding Numeracy

Understanding number requires the ability to determine the total number of objects and reason about them using number relationships.

Understanding Numeracy

- **Flexibility:** The ability to look at numbers in a problem and consider more than one way to solve the problem. They develop the ability to see relationships in the numbers more clearly.

Understanding Numeracy

- **Accuracy:** Getting the right answer is not the only part of numeracy that matters. Students should begin to look for strategies that work for certain numbers: friendly numbers.
- **Efficiency:** Knowing which strategy may be used to solve the problem.

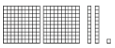
Understanding Numeracy

Well-meaning teachers have often worked against students early intuitive number sense by emphasizing procedures and sacrificing the development of number relations.

Understanding Numeracy

People with good number sense capitalize on the own natural math sense despite the emphasis placed on procedures in most schools (Stein, 2008).

EXPLORING PLACE VALUE



Reflect

- Why is the concept of place value difficult for students?

Understanding Place Value

- Why is place value so difficult?
  - Unitizing:
    - 34 is 34 ones and 3 tens and 4 ones
    - One ten and 10 ones simultaneously
  - The idea of zero--is conceptually different from all previously developed numbers in that it is not connected to real objects.
    - Students must understand that when a group of ten is completed the symbol for zero appears as a part of the written numeral: for example, the number 10.
    - Absence for objects
    - A number of its own, defined mathematically as n-n

What is Place Value?

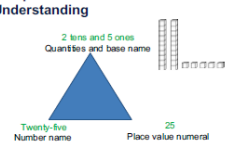
- Place value is the key to teaching computation with our base-ten numerals (Ashlock, 2006).
- Place Value refers to the social-conventional knowledge that in 333 the first "3" is three hundred, the second "3" is thirty, and the last "3" is three.
- It is common for students to think the "1" in 16 means "one" until about fourth grade (Ross, 1986)

SEE SAW

Representing Sets of Tens and Ones

- Number Bags

Components of Place Value Understanding



The 1988 *National Assessment of Educational Progress* results showed that less than one half of third grade students were successful with place value tasks beyond tens, which is related to students understanding of addition and subtraction algorithms.

Representing Whole Numbers

- Empty your bag of straws.
- Bundle your straws so that you could efficiently determine the total number of straws that you have.
- How could you use skip counting and counting-on to determine the combined value of the set?

Representing Whole Numbers

- Use the strips of paper to group the counters into as many equal sets as possible.
  - Set any leftover counters next to the strips of paper
- How did you group your counters?
- How could you use skip-counting and counting-on to determine the value of your set of counters?

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Representing Whole Numbers

- Group the counters into sets of tens and ones on the ten frame mini-mats.
- Use tens and ones cards to describe the set of counters.

Representing Whole Numbers

- Organize the linking cubes on a Tens and Ones Work Mat.
- Use Tens and Ones Cards to describe each sets.
- Use pictures of linking cubes to represent each set of counters.
- Record the combined value of each set.

Understanding Place Value

- **Canonical representations:** No more than 9 objects may be used in any position.
- **Non-canonical representations:** Allow more than 9 and are necessary when representing multi-digit computational algorithms (Carrying and Borrowing).

Multiple Representations

56-38

A study by Cauley (1988) found only 34 out of 90 second and third grade students understood that the number 56 had the same value before and after they borrowed.

Representing Whole Numbers

- Use the place value spinners to generate hundreds, tens, and ones.
- Use bundling straws to represent the 3-digit number generated then organize the straws on the Number Organizer.
- Use the pictures of straws to complete the Picture A recording sheet.
- Record the values for Picture A on the Building Straws recording sheet.
- Repeat for Pictures B and C.

Multiple Representations

- Use the pictures to record a model for Picture 1.
- Record the information for Picture 1.
- Redistribute the counters to create two *different* models of the *same* number and record the information on the recording sheet.

Representing Whole Numbers

- Base 10 Blocks
  - It is important for students to understand that there are efficient ways to represent numbers such as representing the number 258 with the least number of base ten blocks possible, which corresponds to the way in which we write numbers in standard and word form.
- Tens and Ones Mat
- Hundreds, Tens, and Ones

Representing Whole Numbers

- Trace 1 hundreds flats onto the Grid Paper.
- Trace 2 tens rods onto the Grid Paper.
- Trace 4 ones units onto the Grid Paper.

Representing Whole Numbers

- Use base ten blocks to represent the number 243 on the Place Value Mat.
- Use a different color to record the total value of each digit on the Number Strip.

2	0	0	+	4	0	+	3

Naming Whole Numbers

- Use the place value spinners to generate hundreds, tens, and ones.
- Use base ten blocks to create a model to represent your number.
- Select the pictures cards needed to represent the number of hundreds, tens, and ones in your model and organize the cards on the Our Number Organizer.
- Record a picture of your number on Naming Our Number.

Naming Whole Numbers

- Dimes and Pennies

Comparing and Ordering Whole Numbers

Naming Whole Numbers

- Select the words cards needed to represent the value of the hundreds, tens, and ones in your model and organize the cards on the Our Number Organizer.
- Use words to describe your number on Naming Our Number.
- Select the numbers cards needed to represent the value of the hundreds, tens, and ones in your model and organize the cards on the Our Number Organizer.
- Record an addition sentence that represents your number on Naming Our Number and find the sum.

Naming Whole Numbers

394

Comparing Sets of Tens and Ones

- Use the linking cubes to create a "train" for each number on the recording sheet.
- Regroup each train, into sets of tens and ones.
- Using the pictures of linking cubes, complete the recording sheet.
- What questions could we use to debrief this activity with our students?

Comparing Sets of Tens and Ones

- Cut out the Comparing Picture Cards
- Choose two picture cards and one word card.
- Arrange the cards so that the words correctly describe the relationship between the two picture cards.
- Repeat the process until all cards have been used.
- Fill in the missing information on the picture cards.

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Using Place Value to Order Numbers

- Use the Digit Cards to create 3 different 2-digit numbers.
- Arrange the numbers in order from greatest to least on the Ordering Numbers recording sheet and then complete the comparison statements.
- Draw pictures on the Proving the Order recording sheet to verify the orders of the 2-digit numbers.

- How do the activities we have explored align to the Texas Essential Knowledge and Skills (TEKS)?

Patterns in Place Value

- Use base ten blocks to complete the Patterns in Numbers recording sheets.

Patterns in Place Value

- Place a counter on the number 146.
- Move the counter to the new location described.
- Record the new location (number) on the Patterns in Numbers recording sheets.
- With your table group, discuss the patterns you noticed.

- The base ten number system provides an efficient way to represent numbers.
- The 10 different digits can be arranged to express any whole number.
- Students must understand that the base 10 number system allows you to create a new place value unit by grouping ten of the previous place-value units. For example 10 ones is 1 ten. 10 tens is 1 hundred. 10 hundreds is 1 thousand.

Investigating Students' Thinking

- Task: Write the number that is ten more than 47.
  - Student A: Writes 48 quickly.
  - Student B: Writes 57 immediately.
  - Student C: Writes 50 promptly.
- What do these responses tell us about each student's understanding of place value concepts?
- What other question might we ask to better understand these students' thinking about place value?

Investigating Students' Thinking

- When asked to write a number one more than 342, half of the third grade students made a mistake.

  453, 443, 452, 1,342, 5,342
- When asked to write a number 10 more than 243 about two-thirds were incorrect

  233, 353, 342, 251, 1,243, 121,413

Patterns in Place Value

- Use the 0-999 Chart to complete the activity Find the Numbers.

Reflect

- Why might a student struggle with comparing and ordering numbers?

Comparing and Ordering Numbers

- Use the number cards to complete the Order Up! recording sheet.

Comparing and Ordering Numbers

- How could you use an open number line to compare and order whole numbers?

- It is important that students compare numbers based on place value.
- Students should understand that 89 is less than 98 because the digit in the tens place has a value of 80 in 89 and the value of the digit in the tens place is 90 in 98.

Comparing and Ordering Numbers

- Each person drops a counter on the 0-999 Chart.
- Use the numbers to record the information needed on the Making Comparisons recording sheet.
- Repeat the process.

- How do the activities we have explored align to the Texas Essential Knowledge and Skills (TEKS)?

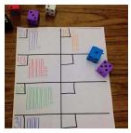
Game Plan

- What should developing place value concepts look like in my classroom?

APPENDIX B

PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Could a student play this game and not understand place value? Why or why not?



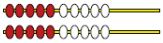
- By fourth grade, students are expected to interpret the value of each place-value position as ten times the position to the right and as one-tenth of the value of the place to its left.

555

500 (5 x 100) 50 (5 x 10) 5 (5 x 1)

Making Connections . . . Facts

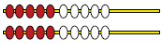
- Make a Rekenrek



- What do you observe about the rekenrek?

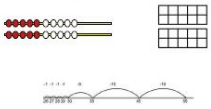
Making Connections . . . Facts

- Facts with a Rekenrek



- Show 5 + 6. What do you notice?
- Show 7 + 7. What do you notice?
- Show 8 + 6. What do you notice?
- Show 13 - 5. What do you notice?

Addition and Subtraction

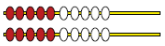


Understanding Place Value

- Place value should be taught in conjunction with addition and subtraction
  - 9 + 6
  - 9 + 1 is 10 so 1 minus 6 is 5
  - 10 + 5 is 15

Making Connections . . . Facts

- The Birthday Party



Making Connections . . . Facts

- Explore the use of Double Ten Frame for addition and subtraction facts.
- Explore the use of the little Ten Frames for addition and subtraction facts.
- Ten Frame Cards

Facts

- Number Cards (Fast Facts)

Reflection: Addition/Subtraction

- How do students use place value when solving two-digit or three-digit addition and subtraction problems?

Addition and Subtraction

How would you solve these two problems?

- 272-14
- 283-275

Compare these approaches

686 + 78

Add 6 and 8, write down a 4, carry a "1"
Add 8 and 7, write down a 5, carry a "1"
Add the 1 and the 6

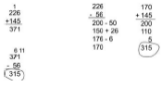
600 + (80 + 70)
600 + 150 + (6 + 8)
750 + 14
764

Algorithms

- If the traditional algorithm is the students' only computational strategy then the algorithms' digit approach may inadvertently affect students progress in higher mathematics.
- In a study of 44 mathematics Dowker (1992) found that mathematicians tend to use an understanding of arithmetical properties and relationship rather than school taught techniques.
  - Involved enjoyment: playing with numbers
  - Only used traditional methods about 4% of the time.

Compare these approaches

Ms. Jones had 226 yellow pencils and 145 red pencils. Ms. Jones gave Mr. Garcia 56 of the pencils. How many pencils does Ms. Jones have now?



- Do both of these students have an understanding of place value? Why or why not and how do you know?

Student Errors

- Review the following student errors in the Ashlock book (pp. 100-106).
  - Gary
  - Mike
  - Mary
  - Carol
  - Dorothy
  - Cheryl
  - George
  - Donna
  - Barbara
- Don't forget to check your answers!

Student Errors

- How are these common errors similar/different to errors you have seen in your classroom?

APPENDIX B

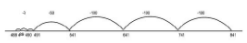
PROFESSIONAL DEVELOPMENT POWERPOINT® SLIDES

Mental Arithmetic

- Mental arithmetic does not mean solving the problem in your head. Instead, it means you are using your head to reason and you may use pencil and paper to keep track of your mental steps.

Flexible Strategies

- The Open Number Line
  - Introducing the open number line using linking cubes



Addition and Subtraction

Sharing gives students opportunities to hear new ideas and compare them with their own and to justify their thinking.

NCTM Principles and Standards (2000), pg. 198

Addition and Subtraction

- Participate in a Round Robin.

Flexible Strategies

- Place Value Splitting Strategy
  - The individual digits 4 and 3 represents a partitioning of the whole into a "tens part" (40) and a "ones "part" (3), which is equal to the whole, 43.

Flexible Strategies

- Practice using these strategies to solve word problems.

Flexible Strategies

- Using the 100's Chart to add and subtract.

Game Plan

- What should addition and subtraction look like in my classroom?

APPENDIX C

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT A

Session 1 Observations	Reflections
Students working in small groups (Teacher to small group). • T: Showed the number 52 and asked students to use base ten blocks to build the number. • S: Used longs and units to show 5 T and 2 O. • T: Handed each student 1 hundreds flat. • T: What do you notice about the hundred's flat? • S: It feels squishy. • S: It is the same size (comparing heights of both). • T: Compare the length of 1 ten and 1 hundred. How many tens will it take to fill up a hundred? • T: How could you use base ten blocks to build the number 123? Where would we put these on our H-T-O Mat? • S: The flat in the hundreds, the longs in the tens place, and the units in the ones place. • T: How could you use skip-counting to prove you represented 123? • S: One hundred, one hundred and ten, one hundred and twenty, one hundred and twenty-one, one hundred and twenty-two, and one hundred and twenty-three. • T: Remember when we are saying a number we do not say "AND," we say one hundred twenty . . . • Process and questions were repeated for the numbers 231 and 165.	• Teacher is trying to build understanding conceptually. Are students relating the value of the tens and ones or just using 5 rods and 2 ones? • Have the students previously compared longs and units? *Yes* • Is the H-T-O Mat with the double ten frame helpful for the students? *The mat has helped the students keep their ones organized. There are still some students that count the units individually instead of recognizing the ten-frame as a ten because they did not have experience with a ten-frame or double ten-frame.* • Teacher understands the meaning of the word "AND" as a decimal. • How did the students do in comparison to what you had expected? *Most of the students did OK, but I discovered that some students have no idea how to skip-count.* *Also, I have some students who still do not understand that a long means 10.* Additional Post-observation Conversations: For students struggling to see the "long" as a representation of 10, build numbers with a variety of concrete representations. • Unifix© Cubes • Bundling straws and rubberbands • Counters and mats

S: Student
T: Participant A

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT A

Session 2 Observations	Reflections
Students working in small groups. T: Noticed dice flying through the air and redirected the group. What are you working on? • S1: We are playing a game and rolling dice to make a number. • S2: I am winning. • T: Remind me how do you play this game? • S2: We each roll a dice and we have to write the numbers down in any order then see who has the biggest number. • S1 rolls a three, S2 rolls a four, and S3 rolls a 6 then S1 recorded 346, S2 recorded 643, and S3 recorded 634. • T: Who won this time? • S1: I did. • T: What number did you make? • S1: 346 • T: S2 and S3, did you make a number less than or greater than 346? • S2: Greater; mine is the biggest. • Which digit is in the hundred's place? What is the value? • S1: 300; S2: 600; S2: 600 • S1: So they both win. • T: Let's look at the tens and ones place. Does the tens place in both of the numbers have the same value? • S1: Oh, one is 40 and one is 30. The number with the 4 in the tens place is the biggest. • T: If the digits in the tens place were both four and have a value of 40, where would we need to look? • S3: We would need to look at the ones place to see which one was the greatest. • S2: Let's play again!	• Teacher redirected students by asking them questions about the activity. • Did the teacher previously model this activity? *Yes, I modeled the activity earlier before you came in.* • How are students held accountable for work when the teacher is working with other groups? *Most of the time I have them record something in their math journals. For example, with this activity I can look at the numbers to see how many times they played and compare them with their group to see if they were all recording the same digits.* • Students seemed engaged . . . wanting to "play." • Are the groups without the teacher asking questions making connections to the value of the digits? *As I observed and listened to other groups, I do feel like some students were just writing down number without thinking about a strategy for making the greatest number.* Additional Post-observation Conversations: • Allow students struggling with creating and comparing numbers, to build numbers on the H-T-O Mat with base ten blocks. • Add a recording sheet for the values of each digit or a place value chart for students to record digits. • Discussed the progression of moving students from base ten blocks to pictorial representations and numerical representations.

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT A

Session 3 Observations	Reflections
Small-group Instruction (Teacher facilitated small groups.): • T: Use your base ten blocks to make 241 on your place value mat. • T: Use base ten blocks to represent 241 in a different way. • T: What could you do to represent 241? • S: You can remove 1 ten and add 10 ones and "then yeah." • T: What is the value of the set of base ten blocks now? • S: 241 • T: We have been talking about representing numbers in different ways by building them with base ten blocks. Now, we are going to record pictures of our different representations. • T: Represent 456 with pictures on the Same Value, Different Picture recording sheet (From PD). Students in this group built, then recorded multiple representations for 456 as the teacher walked around and looked to make sure students were completing the work correctly. • As the teacher worked with this group, one group of students continued working to build two-digit numbers while the other group was working on daily math routines.	• Teacher connected to students' prior knowledge. • As students recorded the new pictures, they could have recorded different values in a table or some organization to show 200 + 40 + 1, 100 + 140 + 1, 200 + 30 + 11. *Didn't you give us a sheet for recording the values? I forgot about this until just now when you mentioned it.* • What is the plan to scaffold instruction for students who are still working with two-digit numbers? *These students are still struggling with skip-counting and counting by ones. We are skip-counting during warm-up. For the others,we have made sets using the various tools like we talked about and now we have made the connection to 10 ones being represented by 1 ten.* • Additional Post-observation Conversations: • *Some students still saying 2 + 4 + 1; any ideas?* • Record each place's value: 2 H 4 T 1 O 200 + 40 + 1 241 • Provide a calculator to let the students add the values. • Continue skip-counting and counting-on conversations.

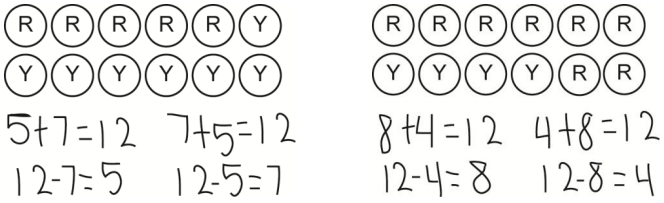
APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT A

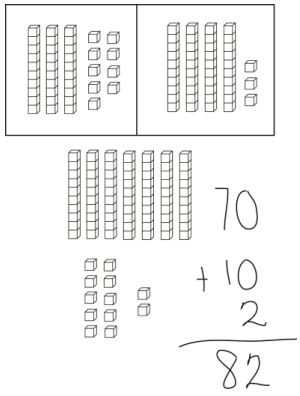
Session 4 Lesson Planning

Planning for incorporating addition and subtraction concepts for struggling students and advanced students:

1. Use a set of two-colored counters to decompose and compose small quantities, and record the corresponding addition and subtraction number sentences. This will also help build an understanding of facts.



2. Use base ten blocks to represent and build two different numbers and then allowing the students to explore a variety of ways to combine the sets in order to determine the total number.



3. Allow students to describe their solution strategies. Ask students to give you their solutions and the various ways they arrived at the solution.

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT B

Session 1 Observations	Reflections
Student tables arranged in groups of 4. T: Wrote the number 10 on chart paper and asked students to tell her what addends they could use to make 10. • S: 2 + 8, 6 + 4, 5 + 5, 7 + 3 • T: (To me) What else can I do or ask? Should I let the students use counters? • The students could use Unifix© Cubes to model the situation. • T: Handed a set of Unifix© Cubes to each student. • T: Use Unifix© Cubes to decompose 10 into parts. • T: What number sentences can I write for 10? • S: Most students decomposed cubes into two parts so number sentences were similar to the previously recorded numbers (2 + 8, 6 + 4, etc.). • T: (To me) What else? • One student had five groups of two so I asked how we might use that representation to write a number sentence. After prompting, the students said 2 + 2 + 2 + 2 + 2. • T: Recorded the number sentences. • I continued to prompt students to decompose the number 10 into more than two parts as the teacher recorded the number sentences. • I asked the students to prove how they know the number was 10 by using counting-on or by the use of a previously known fact. S: Student T: Participant B	• Pre-observation E-mail: *I need help with getting students to explain their thinking.* • The teacher had a lesson planned but chose to take advantage of me being in the room and felt comfortable asking for my help. I helped her use Unifix© Cubes to teach the concept and I was able to model questioning strategies to get the students to explain their thinking as she had asked for in an e-mail prior to my arrival. Additional Post-observation Conversations : • What did you observe your students doing well or struggling with during the lesson? *My students were struggling coming up with number sentences so that is why I asked if I should have started with counters. I tried this yesterday without them but it did not work, but I did not know if it was ok.* • *What should I do next? Is this a worthwhile activity?* It is the beginning of the year, so yes; you need to start by teaching the students how to compose and decompose numbers concretely. However, some students have the hang of it, so you may want to have the counters available and allow students to use them if they need or want to. Additionally, when you give them a number such as 10, we want the students to think flexibly such that 12— 2 is equal to 10 or 4 + 4 + 4 is equal to 12. The purpose of this activity is to get the students to think flexibly about the numbers as well as be able to explain their process or strategy. • *Next time you come, could you do a lesson on building bigger numbers? I would like to use the lesson you talked about during the professional development, with the mats.*

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT B

Session 2 Model Lesson	

• I provided a model lesson in which the students worked in groups and counted and bagged 36 two-colored counters.
• I asked groups of students to first prove how many two-colored counters were in their bag.
• Most students counted one by one.
• I handed the students several mats (strips of construction paper) and asked the students to place equal groups of counters on the mats and place any leftovers on the side. Some students grouped the counters by twos, tens, fives, and even sevens.
• I asked students to skip-count and count-on to prove there were 36 counters. I purposely saved the groups of 10 until last and asked students which was the easiest to skip-count and prove. Then I asked if anyone wanted to regroup his or her counters.
• All of the students were given an opportunity to regroup their counters into group of tens and some more.
• We then labeled our arrangements as 3 tens and 6 ones using sticky notes.
• We discussed the value of the 3 tens as 30, the value of the ones as 6, and the combined value as 36.
• We repeated the process with the number 42.
• I asked students a final reflection question for their journal: How might it be easier if we have a tool or counter that was already grouped into tens and ones?
	Additional Post-observation Conversations :
	• *Can you explain why you started with this lesson?* Too often, we give the students base ten blocks and assume they understand that they represent "10." Through making it difficult to skip-count, the students generate the idea for themselves that it is more proficient to group objects by tens.
	• *What do I do next?* Students are going to need additional practice creating sets of tens and ones, which is a concept they should have developed in first grade.
	  • Give the students another number such as 57 and ask the students to represent the number using Unifix© Cubes and the mini-ten frames.
	  • The mini-ten frame mats will help the students organize the groups of ten.
	  • Have the students label the number of tens and ones.
	  • Discuss the value of the tens (50) and ones (7).
	  • For the next step, prompt the students to link the ten Unifix© Cubes to make trains.
	  • Explain that there are still 5 tens and 7 ones and still represents a value of 57.
	  • Ask students if the Unifix© Cubes were easier to keep organized/grouped than the individual counters.

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT B

Session 3 Observations	Reflections
• T: Taped a yellow line of masking tape in the hallway and gave students various numbers from 0 to 20 on an index card. • T: Who has the 20? Where is your place on the number line? • S: Stood towards the end of the line. • T: Who has the zero? Where is your place on the number line? • S: Stood at the beginning of the line. • T: Continued the process of calling up students and asking them to stand on the number line in the following order: 10, 5, 15, 1, 19, 7, 11, 16, 4, 9, 15, 18, 2, 8, 12, 17, 3. • T: Asked the following questions: ◦ Is ____ in the correct location? Why or why not? • T: Do we need to make any adjustments or move a student in order to add the new number? • S: Made adjustments as needed. Two students did not think that the number should be adjusted (i.e., 10 should not be in the middle).	• Why did you choose to do a lesson on the concept of number lines? Is this at the rigor specified by the scope and sequence? *My team is comparing numbers on hundreds charts and I wanted to do this first then later move on to the hundreds chart since we talked about the importance of using an open number line.* Additional Post-observation Conversations: • What lessons have you planned in regards to continuing to develop students' understanding of representing two-digit numbers? *We are confused; my team does not think we are working on place value this year because we were told that we have to develop number sense. That is why we did the decomposing numbers activities. My team is using the hundreds charts to compare numbers. Is this right? What do I need to do next?* • Let me get some clarification on the district expectations (scope and sequence) and let us meet during your planning time tomorrow and see what we can map out. **I called Dr. Johnson and asked clarifying questions about the district's expectations (results found in the following planning session). • Extend the number line for numbers to 100. There does not need to be a card for all numbers 0–99: make sure to have the important benchmarks (0, 25, 50, 75, 100, etc.) and make sure that students know that the whole numbers on a number line are equally spaced (3 is the same distance to 4 as 26 is to 27). • Ask questions about place value: What is the value of the 2 in the number 24? Would it come before or after the number 25? Why?

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT B

Session 4 Lesson Planning

I talked to Dr. Johnson briefly to clarify the district's intent of saying the focus is on number sense. When you are teaching place value, you are developing number sense. Like when we talked about 54 being 5 tens and 4 ones or 4 tens and 14 ones, that is a part of number sense. You were developing this understanding with the composing and decomposing activity you did. You will just need to continue this concept as you develop place value understanding for two- and three-digit numbers.

T: *Where do I start? There are so many ideas in my head.*

- I would start by trying the lesson with the Unifix© Cubes. This idea of allowing students to compose and decompose a set of objects in this manner is going to help students when you begin to explore addition and subtraction with regrouping.
- Connect the students' understanding of grouping the Unifix© Cubes into sets of tens and ones with the base ten blocks and explain how these are already prebundled. You can look back to the Number Bag: Representing Whole Numbers activities we explored during the professional development.
- As students build numbers with base ten blocks, make sure you connect the values. For example, when you build 64, make sure the students just do not count out 6 tens and that connect this to the idea that the 6 tens represent a value of 60. You will also want students to connect the value of the model to the expanded notation (60 + 4). You can look back to the Number Strip and Naming Whole Numbers activities we explored during the professional development.
- To build on the composing and decomposing, you will need to show the students that bigger numbers can also be composed and decomposed into multiple representations (6 tens and 4 ones or 5 tens and 14 ones). As you create multiple representations of numbers, continue to make sure the values are connected and recorded. You can look back to the multiple representation activities we explored during the professional development.
- Next, you will need to work with comparing and ordering numbers using the sets of base ten blocks. Eventually you will need to wean them off the models and move to pictorial representations, including a number line to compare and order models. Finally, you will move them to abstract representations. During the professional development, we explored several activities: Comparing Sets of Tens and Ones, Using Place Value to Order Numbers, Patterns in Place Value, Comparing and Ordering Numbers, etc.

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT C

Session 1 Observations	Reflections
Students first worked individually to answer these five questions in the Math Warm-up. 1. What is another way to make 25? 2. What is another way to make 71? 3. What is 60 + 500 + 8? 4. What is 1 one, 5 hundreds, and 0 tens? 5. How could you figure out the sum of 4 + 4 + 4? • T: Look at your elbow partner's work and talk about how you solved these five problems. • Students talked to their partner for approximately five minutes to discuss their responses to these questions. • T: What did you get for number one? • S: 20 + 5, 23 + 2, 10 + 15, 0 + 25, 25 – 0 • T: What did you get for number two? • S: 70 + 1, 50 + 21, 72 – 1, 69 + 2 • T: What did you get for number three? • S: 658 • T: What did you get for number four? • S: 501 • T: How did you figure out the sum? • S: I skip-counted by fours: 4, 8, 12. • S: I added four and four, which is eight; then I added four more, which is 12.	• I had not thought about the value of mixing the order of the hundreds, tens, and ones and requiring the students to reorder the values in order to solve the problem. • Is this a district-wide warm-up/review resource? *I am not sure if other campuses use it, but we all use Target Math on our campus.* Additional Post-observation Conversations: • *I wanted to share that I tried what you suggested during the professional development; I am trying to let my students do more of the talking. I was shocked they could actually use what they learned about place value in first grade and apply those understandings to solving an addition problem in a flexible manner. My goal is to get the students more efficient at communicating about mathematics: how they solved a problem or how they know their answer was correct.* • This is something that will take time for the students to become proficient at, and it will require the use of teacher modeling to help facilitate the process. Sometimes it is hard for us to be quiet and let the students talk; however, we can still talk by asking questions to help students explain and/or extend their thinking.

S: Student
T: Participant C

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT C

Session 2 Observations	Reflections
5E Lesson: Multiple Representations of two-digit numbers Engage: T: Showed a model of 6 longs and 3 units. • T: In your journal, create a pictorial model of this representation then explain how you found the value. • S: Drew pictures and responded to the question in their journals. Explore: T: Each student was given a baggie with base ten blocks. • T: Is there another way we can show 63 in a way other than 6 tens and 3 ones? Work with your table group to try to answer this question. • S: Worked to represent the number 63 in a different way. • Explain: T: What do you notice about the base ten blocks? • S: We used a lot more units this time. • T: How do you know the value of your set was still 63? • S: I skip-counted 10, 20, 30, 40, 50, 51 . . . 63. • T: Looking at your base ten blocks, what does 10 units equal? • S: 1 long or 1 ten • T: Is there another way to represent the number 63? • S: We used 63 ones. • T: Is it easier to see the value of the base ten blocks when they are organized into tens or all of the ones? Why? • S: The longs are easy to count because you can count by tens and not by ones. • T: How could we create 63 with the fewest/least number of base ten blocks? • S: 6 tens and 3 ones	• Students must have previously connected the concrete models to pictorial models. *Yes, we have been practicing building and recording.* • Used skip-counting as modeled during the summer. Was this previously used while teaching place value? *No, I had not thought about using skip-counting and counting-on to help the students find the total value of their set. This has helped my lower students who cannot quickly add the tens and ones.* Additional Post-observation Conversations: • As students recorded their pictorial models, did they record the corresponding values of the tens and ones? *No, but I guess we could have added that step.* • Values may be record as numbers, words, and expanded form (i.e., sixty-three, 63, 6 tens 3 ones, 60 + 3). This can be done for the various representations of 63.

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT C

Session 3 Observations	Reflections
5E Lesson: Multiple Representations of Two-Digit Numbers—Continued Elaborate: T: Distributed base ten blocks. • T: You will work with your table group for this activity. Each of you will use your base ten blocks to build a two-digit number. Go ahead and build your number then wait for directions. • S: Students used base ten blocks to create various numbers. • T: Everyone stand up and move to your neighbor's seat. Now, see if you can determine what number your neighbor created. • T: Once you think you know, check with your neighbor to see if you are correct. • T: What number did you neighbor represent? • S: Shared different two-digit numbers. • T: Now, take your neighbor's set and see if you can represent the same number in a different way. You may have to use more or less longs or units; just remember, the total values just need to be the same. • S: Worked to create a different representation of the number. The teacher had to remind students it was ok to "trade" base ten blocks. • The process repeated as students moved around the group, creating a total of four representations of each number. Evaluate: T: Gave each group one piece of manila paper. T: Prompted students to write a two-digit number in the center of the paper. T: Work together to draw four pictures that represent the value you recorded in the middle. (See sample student work.)	• Students seemed to easily decompose the numbers. *I think the questions in our daily warm-up helped prepare the students for this idea.* • Student management of moving seats was easy for students to follow. • How were students able to perform on the Evaluate? *Let us look at their work. You can see on some there is erasing. I think there were students in each group who knew how to do it so I think they made sure the paper was correct. As you heard, the students talked to each other about their thinking; that was the goal and purpose of this Evaluate. The goal was not to evaluate each individual student.* Additional Post-observation Conversations: • How might some type of recording sheet enhance the Elaborate? What if you used the multiple representation chart you received during the professional development. *Well, we did not use a recording sheet because we did not want the students to be confused with expanded notation. We are going to do expanded notation later when we teach bigger numbers.* • For the Evaluate, you could ask the students to record the value of the tens and ones. You cannot overdo connecting the value to the models. *Will that confuse them since it is not expanded notation?* • Expanded notation does not have to necessarily all end in zeros. When students record 40 + 8 and 30 + 18, they are recording expanded forms. • How many tens? How many ones? • What is the value of the tens? Ones? • What is the total value of the tens and ones?

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT C

Session 4 Observations	Reflections
Using Flexible Strategies • T: What answer did you get for 211 + 303? Thumbs up when you are ready. • T: What answer did you get? • S: 514, 500, 305, 113, 114 • S1: I got 514 because I did 3 + 2 = 5, then I put two zeros on them to get 500. 11 + 3 = 14 so 500 + 14 = 514. • T: Could you have added 200 + 300 right away? • S1: Yes! • S2: I also got 514. I added 200 + 300 = 500, 1 ten and no tens in 303 is equal to 10, and 1 + 3 = 4, so 500 + 10 + 4 = 514. • S3: I got 305. I got 30 from the 3 and 30 + 5 is equal to 305. • T: Where did you get the 5 from? • S3: I don't know. • T: Did you hear how S1 and S2 explained their answers? Does their information help you in any way? • S3: Not really. • T: OK, we will work on it a little more later. • S4: I got 304 because 3 is after 4; it is next. • T: So, what did you do with the 211? • S4: I did not know I was supposed to add. I get mixed up about that as a kid.	• All of the answers were recorded without indication about which answers were incorrect or correct. • How do you plan to address those students who did not understand how to get the correct answer? *I plan on working with these students on smaller numbers and continuing to work on place value. As you can tell, one did not even know what he was adding and the other did not recognize the value of 211.* Additional Post-observation Conversations: • How does your team plan mathematics? *We meet once a week as a team to plan. Our AP** meets with us to help us figure out what the scope and sequence is telling us to do. Since we are following the new TEKS, we do not have a lot of resources, so we are having to make up lessons and activities as we go.* ** The AP also attended the 12 hours of professional development and took the LMT. She received the highest score out of all of the participants. She was previously a middle school mathematics teacher. • Are you using any of the materials from this summer's professional development? *Yes, we are using the activities, and we are using the ideas to springboard into 5E lessons or other activities.*

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT D

Session 1 Observations	Reflections
Building Two-Digit Numbers with Base Ten Blocks T: Build a two-digit number, then stand. S: What is a two-digit number? T: A number between 11–99. T: Rotate to the next desk. Figure out what the person made. S: Puzzled expressions T: Look at the back wall and start walking. S: Students took an extended amount of time to build a two-digit number. Blocks were falling on the floor and some students were not creating a number. S: Tell the person next to you what number they made. T: Go back to your seat and make sure you have 50 units and put them back in the bag. T: How can we count these quickly . . . 5's, 10's? S: Every student counted units by ones. Some students had extra and some students did not have enough. Students were recounting the unit cubes by ones.	• How long have you been building numbers with base ten blocks? *We have been building numbers for about a week.* • Is there any type of accountability for each student to complete the task of building a number? *I just walk around and check.* • How do you formatively assess your students? *As I walk around, I check to see if the students can build the number.* Additional Post-observation Conversations: • Connect to the value of the tens, ones, and the total value. • Accountability recording sheet: The value of the tens is ____. The value of the ones is ___. The combined value is ____.

S: Student
T: Participant D

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT D

Session 2 Observations	Reflections
Building Multiple Representations of Two-Digit Numbers •T: Wrote 52 on chart paper. •T: How could we make 52? •S: Several students simply counted out 5 longs and 2 units. •T: Prompted students to try to create 52 in various ways. •S: 4 longs, 12 units; 2 longs 32 units •T: What else? What about 5 longs? Two longs? •S: 3 longs, 22 units •S: One student appears to be asleep. •T: If we use one long, how many units? •S: 52 •T: What is the easiest way to represent it with base ten blocks? •S: 5 tens and 2 ones	• Was this the first day of instruction? *No, we practiced this idea yesterday.* • Are students able to make a connection between the value and the base ten blocks representation? *They should know that it is 50 and 2.* • Does making sure students list every combination take away the goal of developing flexible thinking? What are your thoughts? *I want to make sure we list all of the possible combinations each time.* Additional Post-observation Conversations: • Is there anything I can help you with as far as resources or activities? *Not really, I just do what my team tells me to do.*

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT D

Session 2 Observations	Reflections
Multiple Representations of Two-Digit Numbers • T: Each student was given a baggie with base ten blocks. • T: Are these all the ways we can represent the number 63? • S: Some students worked to represent the number 63 in a different way and some were off task. • T: What are the other ways to represent the number 63? • T: Recorded the ways in order on the chart paper. • S: 6 tens, 3 ones; 5 tens, 13 ones; 4 tens, 23 ones; 3 tens, 33 ones; 2 tens, 43 ones; 1 ten, 53 ones; 63 ones • T: Repeated the process for the number 74.	• Was this part of the 5E lesson that Participant C's class was doing? *Yes, this was part of the lesson.* • Which phases have you completed and which phases will you complete in an upcoming lesson? *We have been practicing most of these parts in my classroom already so we just worked on the practice part of the lesson: the Explore phase. We will be completing the assessment piece tomorrow.* Additional Post-observation Conversations: • Do you usually teach lessons in the 5E model? *The lessons that my team writes for us to follow are in the 5E model.* • Do you feel the 5E model is valuable? *For the most part, it just takes a long time to go through the process.* • Have you had any professional development or training on the 5E model? *Not really, we just have to use it.*

APPENDIX C

INSTRUCTIONAL COACHING: PARTICIPANT D

Session 3 Observations	Reflections
Make It 4 Ways T: Provided each student with a piece of construction paper with the numbers 38 recorded on the front and 57 recorded on the back. T: Work quietly at your seat and create four different pictures to represent each of these two-digit numbers in a different way. S: Some finished the task quickly and finished before others had even started working on the task.	• Every student was given the same two numbers. *Yes, I gave the students the same number because this was their assessment.* • Was the Evaluate written for individuals or as group work? *It was written for groups, but I wanted to see which of my students could do it. If they work together some students will not do anything.* Additional Post-observation Comments: • Remember how I talked about describing the values? What you could easily do is have the students add the values of each set below their picture. *I know but that gets confusing for the kids because 30 + 18 is not an expanded notation and that is what we are going to work on next. Some teachers were having their students record the values but it was causing confusion. I do not know how this is going to work next week, but we will see.*

APPENDIX D

APPENDIX D

GENERAL FORMAT FOR THE POST-TEST INTERVIEW: PARTICIPANT A

Participant: A	Date of Interview: September 2012
• Which questions were the easiest for you? • Which ones were the most difficult?	The question with subtraction was easy because I was able to work it out. The most difficult was question 15 because I do not know how base ten blocks are later used for decimals.
Looking at the questions on the LMT, how did you reach your conclusion? • Why does this work? • What is happening in the regrouping part? • Does the value of the number change when we regroup? Why or why not? • What does the small number represent?	Question 1: We talked a lot about the variety of representations and this understanding helped me answer this question. I did not notice the first time that one of the questions said "tenths." I did not read carefully.
	Question 15: At first I thought the one (units cube) was the whole then I had to think about the fact that it takes 100 of the units to make a hundreds flat or whole.
	Question 10: If you add 10 to ones place, you can add a value of 10 to the other place (tens) to make the numbers friendlier. As long as you do the same to both the tens and ones place, it works. The small number is the 3 tens, or 30.
	Question 13: This problem could be solved in the three different ways. The first time I thought about place value and subtracting the hundreds, then tens, and then ones. It wasn't until the second time to take the test that I saw the student counted up to find the answer in A and that he or she made friendly numbers by adding 4 and 40 to both numbers to make them friendlier.
	Question 25: This one was hard to figure out because we always say you can't take 9 from 7. However, we forget about negative numbers; once I remembered it was simple to say $-2 + 20 = 18$.
What additional support do you feel you need in regard to place value?	I would like help with differentiating for students who are still struggling to get the basics (counting by 10s, making a number, etc.) I would like more quality activities and model lessons.

APPENDIX D

GENERAL FORMAT FOR THE POST-TEST INTERVIEW: PARTICIPANT B

<table>
<tr><th>Participant: B</th><th>Date of Interview: September 2012</th></tr>
<tr><td>• Which questions were the easiest for you?
• Which ones were the most difficult?</td><td>The easiest question was number 1. I drew a place value chart to help me think about the answers.
<table><tr><td>H</td><td>T</td><td>O</td></tr><tr><td>3</td><td>9</td><td>1</td></tr></table>
Several were difficult. I am not sure if I got them right or not.</td></tr>
<tr><td rowspan="4">Looking at the questions on the LMT, how did you reach your conclusion?
• Why does this work?
• What is happening in the regrouping part?
• Does the value of the number change when we regroup? Why or why not?
• What does the small number represent?</td><td>Question 1: This was discussed above so it was not readdressed.</td></tr>
<tr><td>Question 15: Since the unit is the smallest, it had to represent the hundredths and the flat had to be the whole.</td></tr>
<tr><td>Question 10: I have no idea how this happened. I know that you cannot borrow from the bottom number when you are regrouping so I am not sure how the student ended up with the correct answer.</td></tr>
<tr><td>Question 13: All of these strategies give you the right answer. For some of the strategies, I am not sure really sure why they worked but they did. I can tell one student counted up, the other one used place value, but I really do not know how this one was solved (like Figure 5).
Explained how 4 and 40 was added to both numbers to make it friendlier/easier for the student to solve.
I would have never thought to do it that way.</td></tr>
<tr><td></td><td>Question 25: This one is tricky. I always tell my students you can not 9 from 7, but I guess you can if you use a negative number.</td></tr>
<tr><td colspan="2"></td></tr>
<tr><td>What additional support do you feel you need in regard to place value?</td><td>I need help planning what to do. I have lots of things in my head but I do not know how to fit everything together.</td></tr>
</table>

APPENDIX D

GENERAL FORMAT FOR THE POST-TEST INTERVIEW: PARTICIPANT C

Participant: C	Date of Interview: September 2012
• Which questions were the easiest for you? • Which ones were the most difficult?	Question 1 was the easiest because it is just asking the different ways to show that number with base ten blocks. Question 25 is the most difficult because although the answer is correct . . . I am really not sure how they got it.
Looking at the questions on the LMT, how did you reach your conclusion? • Why does this work? • What is happening in the regrouping part? • Does the value of the number change when we regroup? Why or why not? • What does the small number represent?	Question 1: *This was discussed above so it was not readdressed.*
	Question 15: I just remembered this from my undergraduate classes: base ten blocks can be used for place value of whole number and decimals.
	Question 10: Did I get the question right? *No ☺* I chose this answer because I thought the student just crossed out the wrong one and increased the number instead of decreasing it. *When you add 10 to both the tens and ones place, does the difference between the two numbers change?* No, so I guess it works to add the same amount to both places then
	Question 13: Well, I am not sure why I chose this answer other than I just guessed. I did not have the student here to ask them what they did. *For A, I explained how the student used place value to count up. The numbers on the side are showing the amount they counted up each time. For C, I explained how 4 and 40 were added to both numbers to make it friendlier/easier for the student to solve.*
	Question 25: This question is the most difficult because although the answer is correct . . . I am really not sure how they got it. *What is that in front of the 2?* A minus sign? *What if I tell you it is a negative sign (–2)?* Oh, I get it –2 + 20 is equal to 18.
What additional support do you feel you need in regard to place value?	I really need to see model lessons.

APPENDIX D

GENERAL FORMAT FOR THE POST-TEST INTERVIEW: PARTICIPANT D

Participant: D	Date of Interview: September 2012
• Which questions were the easiest for you? • Which ones were the most difficult?	Question 15 was the easiest because it was logical. (*See explanation below.*) Questions 10 and 25 were equally difficult for me because the students were not able to explain their thinking.
Looking at the questions on the LMT, how did you reach your conclusion? • Why does this work? • What is happening in the regrouping part? • Does the value of the number change when we regroup? Why or why not? • What does the small number represent?	Question 1: Units equal ones, rods are tens, and flats are hundreds
	Question 15: The blocks go in order; units are wholes, rods are tenths, and flats are hundredths. One is a unit, tenths is tens, and there are one hundreds ones in a hundredths. *Which one of these models can be broken into parts and are the parts represented by the other blocks?* So, the unit really can't be broken down so I am thinking that the flat is really the whole that can be broken down into tens, ones, tenths or hundredths.
	Question 10: I am not sure and that is why I chose the "I am not sure answer choice." *For this one, the student added one 10 to the tens places and 10 ones to the ones place before subtracting. You can do this because as long as you do the same thing to both places, the difference between the two numbers remains the same.*
	Question 13: I think that only these two ways were right. I did not think you could solve this problem using the third method. *For C, I explained how 4 and 40 were added to both numbers to make it friendlier/easier for the student to solve.*
	Question 25: I cannot figure out why and there is no explanation shown so I said I am not sure. *What is that in front of the 2?* It looks like a subtraction sign. *What if I tell you it is a negative sign (–2)?* We do not do that in second grade. We tell students you cannot take 9 from 7, so that is why you have to borrow.
What additional support do you feel you need in regard to place value?	I need model lessons. Show Me!